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AN INTERNATIONAL REVIEW OF SPECTROSCOPY AND ASTRONOMICAL PHYSICS

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ASTRONOMICAL PHOTOGRAPHIC PHOTOMETRY AND THE PURKINJE EFFECT. II[1]

By F. E. ROSS

ABSTRACT

Photographic photometry of stars.—The four methods in use are outlined and the principles underlying each are pointed out. The *turbidity method* depends on the increase of size of image with intensity. The Greenwich formula connecting the diameter of the image with the star's magnitude is shown to fail for images less than 50 μ in diameter and a new formula which holds for all cases is suggested: $\sqrt{(d+h)} = a+b \log I$, where a, b, and h are constants. On the assumption that the growth of the image is due to diffuse scattering and reflection of the light, formulae for the intensity of this light at any distance from the edge are deduced and sections of images are given which show both theoretical and actual equiluminous surfaces (Fig. 1, and Pl. XII). However, it is found by experiment that the rate of growth or turbidity (Δ) is not constant but increases with the intensity, for instance from 9 to 12 μ when the intensity is multiplied three hundred fold. This effect is of importance in accurate measurements, since if it is not allowed for, deduced relative magnitudes will vary with the time of exposure. Data are also given showing marked variations of turbidity with the kind of plate (Table IV) and with the telescope used, but if the optical system is well corrected the optical turbidity should be negligible. The probable error of this method is about $0^{\mathrm{M}}1$ for very sharp images but may be reduced considerably by using plates of greater turbidity and with small inherent irregularity. In the *densitometric method*, which involves the measurement of out-of-focus images, the probable error is small except that due to inherent plate irregularities, so the more these are reduced the greater will be the relative accuracy of this method. The variation of turbidity with wave-length, the *photographic Purkinje effect*, is of great importance. The results obtained are reviewed, but because of the complexity of the effect no general conclusions can as yet be given. An explanation is suggested for Abney's discovery that for some plates the least gradation is for the wave-length of maximum sensitivity.

Transmission of some photographic plates to white, blue, green, and red light is given (Table I). While Block and Renwick have found that the absorption is not exponential, for very thin films the law (Bouguer's) may hold, at least approximately.

Photographic turbidity for spectrum lines is found to be the same as for circular images of the same width.

[1] Communication No. 150 from the Research Laboratory of the Eastman Kodak Company.

There are in use at present four methods of astronomical photographic photometry. These are, in historical sequence:

A. Measurement of diameters of stellar images.

B. Measurement of densities or degree of blackness of out-of-focus images.

C. Eye comparisons of blackness and size of image with a sequence of images of known light value.

D. Measurement of total obstructance of the photographic image to radiation, as in the thermoelectric method of Stetson.

The usefulness of any photometric method depends primarily upon its sensitivity, which mathematically can be defined as the ratio of effect to stimulus. In the case of method A, the sensitivity is the ratio of the increase in diameter of a focal image to the logarithm of the exposure, called "astrogamma." In method B, it is the ratio of the increase in density to the logarithm of the exposure, usually called "gamma" or gradation. In D it is the ratio of the increase in the mass of exposed and developed grains in a focal image to the logarithm of the exposure. C is essentially the same as D, but is limited in practice to faint images. Method A depends for its efficacy on the sidewise scatter of light in the emulsion. Method B depends largely on the depth penetration of the light. Methods C and D depend upon a combination or addition of A and B, and accordingly make use of the *total* action of the light, which A or B do not. Their sensitivity is therefore higher and they must moreover be considered more logical methods than A or B.

Distribution of light in the emulsion.—The increase in density of a photographic image and the increase in size are phenomena partly of common origin. The increase in size of an image in the case of a perfectly sharp image is due to multiple reflection, refraction, and diffraction of the image-forming light by the grains of silver halide, which, in thickly packed layers, constitute the emulsion. The light is thus deflected from its original direction perpendicular to the plate in a succession of steps until its direction of flow, measured by normals to the equiluminous surfaces, lies in the plane of the emulsion perpendicular to the geometrical edge of the image. The equiluminous surfaces $A \; . \; . \; . \; . \; N$ (Fig. 1) which lie under the image itself are parallel to the surface of the emulsion,

and can obviously be joined or linked with the equiluminous surfaces which lie off the edge. The photometric characteristics of the density of an image, upon which method B is based, are determined, in part at least, by the surfaces *A* . . . . *N*, while method A is based upon the characteristics of the surfaces *A* . . . . *R*.

There are a variety of factors which govern the penetration of light in an emulsion. Among these may be mentioned the size, shape, packing, orientation, and optical properties of the silver halide grain, optical properties of the embedding gelatine, wavelength and composition of the light, and thickness of the emulsion

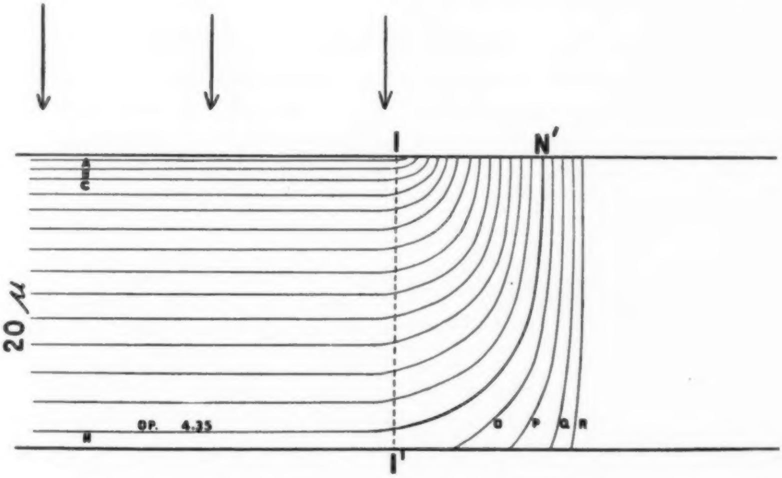


Fig. 1.—Equiluminous surfaces at edge of a sharp image

layer. Concerning the specific action of each of these factors, little or nothing is known, although the subject is of great importance from many points of view.

The silver halide grains which are contained in the high speed emulsion are in great measure in the form of plates or tablets which must be oriented nearly parallel to the surface of the emulsion.[1] Such emulsion would accordingly be expected to show very little side scatter and a high direct or specular transmission. If there

[1] L. Silberstein, "The Orientation of the Grains in a Dried Photographic Emulsion," *Jour. of the Optical Society of America*, **2**, 171, 1921, and **3**, 363, 1921.

were no reflections or diffraction in the passage of the light through the emulsion, the transmission would follow Bouguer's law, which states that equal thicknesses absorb equal percentages of the incident light. The subject has been investigated theoretically and experimentally by Bloch and Renwick[1] who found considerable deviation from Bouguer's law. Calling D the effective density where $D = \log_{10}$ opacity, and putting W = weight of silver bromide in mg per cm^2 they find for a typical fast emulsion:

$$\left.\begin{array}{ll}\text{White light illumination:} & D=0.66\ W^{0.64}\\ \text{Violet light illumination:} & D=1.123\ W^{0.82}\end{array}\right\} . \qquad (1)$$

With Bouguer's law holding, the exponent of W should be unity. A strong color effect is shown, as was to be expected.

For the average fast emulsion $W = 1$ very nearly. Accordingly the corresponding opacities from (1) are:

	Opacity	Transmission
		Per Cent
White light	4.58	21.8
Violet light	13.3	7.5

These values show that the penetration of light in an emulsion is extremely sensitive to wave-length variations. Abney found from photographic tests that an ordinary Kodak film transmitted 22 per cent of the incident light agreeing with the results from Bloch and Renwick for white light.

Values of the transmission of various types of plates have been determined photographically in this laboratory by E. Huse for the three primary colors and for white, with results shown in Table I. The receiving medium in all cases was a Wratten and Wainwright panchromatic plate.

The blue, green, and red in Table I refer to the transmission of Wratten and Wainwright filters C, B, and A respectively. White refers to the acetylene flame screened with Wratten and Wainwright filter No. 79.

[1] "The Opacity of Diffusing Media," *Photographic Journal*, **40**, 49, 1916.

Data on the optical behavior of emulsion film are given by Nutting.[1] The distribution curves of reflected and transmitted light,

TABLE I

TRANSMISSIONS

Plate	White	Blue	Green	Red
	Per Cent			
Seed 30	22	8	39	41
Seed 23	18	8	32	39
Seed process	34	11	40	36
Seed lantern	41	25	43	44
Std. orthonon	13	....	26	30
W. and W. panchromatic	10	5	12	23

drawn from his data, are shown in Figure 2. According to these results approximately 57 per cent of the incident light is reflected and 35 per cent transmitted.

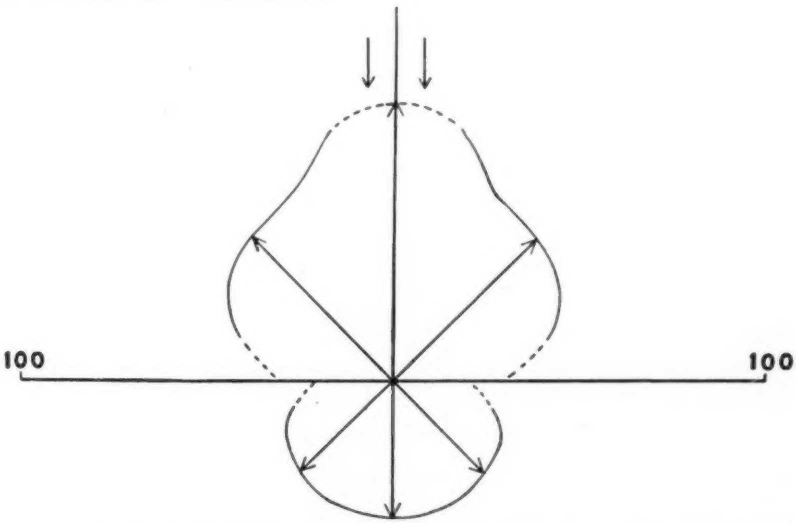


FIG. 2.—Distribution curves (reflection and transmission) of a photographic emulsion, Nutting's data, from visual measurement.

The equiluminous surfaces $A \ldots . N$ (Fig. 1) are drawn to scale from the first of equation (1). The illumination at each surface is 90 per cent of that at the preceding surface, so that each layer

[1] "The Optical Properties of Diffusing Media," *Transactions of the Illuminating Engineers Society*, **10**, 353, 1915.

absorbs 10 per cent of the light incident upon it. It is seen that the effective densities decrease downward in the film, it requiring a progressively greater thickness to produce the 10 per cent drop in illumination. Assuming the total thickness of the emulsion 20 μ, it can be calculated from equation (1) that 38 per cent of the total light absorbed is absorbed in a layer 2 μ thick adjoining the surface, while only 3 per cent is absorbed in a similar layer at the bottom. Assuming Bouguer's law to be the correct one, these numbers become 18 and 5 per cent respectively.

It remains to trace the course of the equiluminous surfaces upon entering the shadow adjacent to the image, where they must rise to the surface. Their course will evidently depend upon the degree of sharpness of the image under consideration. The nearly ideal case of an exceedingly sharp image such as is produced by contact printing will be chosen. It is a simple matter to calculate the light-intensity at any point in the shadow from the rate of spreading of the image, assuming Bouguer's law to hold in this region, which is approximately true, as will be shown later. Thus it is found, assuming light of wave-length 4600 A, that a 10 per cent drop in illumination occurs uniformly in a horizontal distance of 0.71 μ. Scaling off this distance in succession from the point I toward N in Figure 1, the exit points for the complete series of equiluminous surfaces $A \ldots . N$ are obtained. The intermediate portions of the curves lying to the right of II' can be only roughly sketched. To obtain their true form would require an intricate mathematical investigation which would be without profit, on account of the many parameters of unknown relative values.

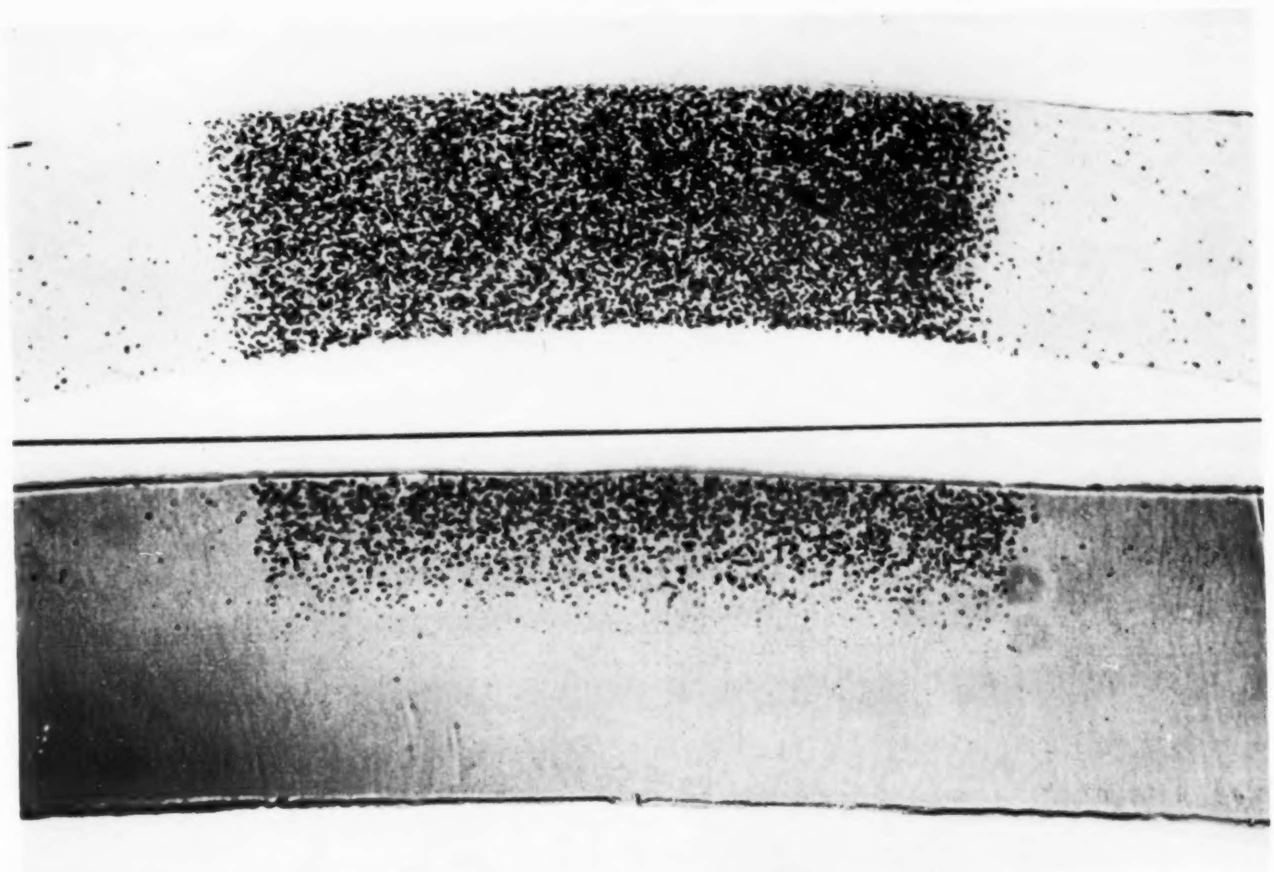
The formula proposed by Tugman[1] for the light-intensity in the shadow adjacent to an image, namely

$$I=I_0e^{-K(x+y)},$$

where x is the distance from the edge and y the distance below the surface, obviously does not hold. A much more complicated relation is necessary. The rate of diminution of intensity downward appears to follow a different law than the law of sidewise drop in intensity. From Figure 1 it is seen that in the ideal case chosen

[1] "Resolving Power of Plates," *Astrophysical Journal*, **42**, 341, 1915.

PLATE XII



Magnified Sections of a Sharp Slit Image
Upper, fully exposed; lower, light exposure

the equiluminous surfaces which lie off the edge of the image are much more closely packed than those lying under the image. This can be explained as a color absorption effect. The light which is traveling parallel to the surface of the emulsion must, on account of its being composed largely of diffracted light, be of shorter average wave-length than the light passing normally through the emulsion. The opacity in this direction will accordingly be higher, resulting in a compression of the series of equiluminous surfaces. In the case of images formed by optical systems, the degree of compression will vary widely with the sharpness or perfection of the image. The horizontal system under the image will be independent of all conditions except composition of the imaging light and its converging angle. For parallel light the opacity should be higher and the penetration less than for light impinging in a wide cone, corresponding to the condition of diffuse illumination. It will be observed that the equiluminous surfaces are shown rising to the surface in a nearly vertical direction. That they actually are of this form in the case of very sharp images seems to be proved by the photomicrograms of an image in which a small amount of spreading has taken place (Pl. XII). The image appears to have expanded as much in the lower layers as in the upper. The section was made while the strips were swollen (with dilute alcohol), the vertical scale being approximately ten times the horizontal. Other cases have been obtained in which the inclination is of intermediate types, corresponding to images of an inferior degree of sharpness. In general it can be stated that the form and orientation of the surfaces of equal illumination are the resultant of the surface distribution due to the optical system and the emulsion distribution due to the spreading of the image-forming light. The problem is thus an exceedingly complicated one. Further difficulties are introduced if the anomalies of development taking place at the edge of an image are considered.

The principles underlying the two methods of astronomical photometry will now be considered. In the densitometric method photometric measurement is limited by the amount of silver which can be deposited in the film perpendicular to the surface, which depends upon penetration of the light and of the developer. When

a long working-range of moderate sensitivity is desired, it is of advantage to have the specific opacity of the emulsion as high as possible and considerable dispersion in grain sensitivity. If, on the other hand, high sensitivity in photometric measurement is desired regardless of a short working-scale, an emulsion of low specific opacity of uniform sensitivity, but rich in silver, is preferable. In the second method of astronomical photometry, in which the increase in size of a small image is made use of, obviously a much wider range of intensities can be measured. The sensitivity of both methods is subject to control in a variety of ways; that of the densitometric method depends upon gamma or contrast, upon developer and development time, and upon the wave-length of light used; that of the size-of-image method depends upon the emulsion, the wave-length of light, but not to any extent upon developer or development time. An additional important factor in determining the sensitivity of the second method is the optical sharpness of the image, which in the case of astronomical observations depends to a great extent upon the steadiness of the air. Perfectly steady air is a detriment to accurate photometric work of this kind, on account of the sharp images, which increase in size at such a slow rate with respect to increase in intensity that a correspondingly low sensitivity is produced. Accordingly a telescope of long focus is preferable in this case to one of short focus, on account of its greater magnification of atmospheric disturbances and, in addition, on account of its optical aberrations, which in general are proportional to focal length (see p. 363).

The writer has reviewed in another place the formulae connecting photographic density and exposure.[1] Astronomers rightly are generally interested in such formulae only to the extent that photographic exposure E involves the stellar magnitude M and exposure time t. This relationship, undoubtedly involving the fundamental law of photographic action, has been given three forms: (1) the reciprocity law; (2) Schwarzschild's law; (3) Kron's law. In order to show that these laws are fundamentally at variance, it is only necessary to point out that according to the reciprocity law, the photographic efficiency is *constant* for all light-intensities.

[1] "On the Relation between Photographic Density, Light-Intensity, and Exposure Time," *Journal of the Optical Society of America*, **4**, 255, 1920.

In the case of Schwarzschild's law, the efficiency ***increases uniformly*** as the light-intensity increases, while in the case of the Kron law, there is an *optimal intensity*, or a light-intensity at which the energy applied has greater efficiency than when applied at any other intensity, greater or smaller. It is thus seen how radically different the three laws are, laws which undoubtedly have root in important physical or chemical phenomena of which we are at present in ignorance. Since they all have been found to be of undoubted applicability in numerous cases, it must be concluded that special circumstances alter the photographic action, throwing it to one or the other of the three *modi operandi*, in a manner which we are unable at the present time to conceive.

The mathematical relation between the diameter of photographic star images and stellar magnitude and exposure time appears to have attracted the attention of astronomers even before the introduction of the dry plate. The earliest formula by Bond in 1857 is:

$$d^2=Q+Pt. \qquad (2)$$

To see how closely this formula fits the modern dry plate, some measures by the writer are plotted in Figure 3, according to this formula. It is seen that the linear relation is far from being realized except for a very short range of 1 to 4 (1.5 magnitudes). Bond notes that the sensitiveness of his plate increased toward the end of the exposure, a fact which appears capable of explaining his comparatively unique formula.

About 1890 a number of formulae applicable to the dry plate were proposed by Scheiner, Charlier, and others, which involved roots of the intensity and exposure time. Among others, the square and fourth root were adopted. As in the case of the Bond formula, these formulae are compared with the same data as in Figure 3. In addition formulae involving the sixth and eighth roots are shown on the same diagram (Fig. 4). It will be noticed that as the root increases the relation becomes more nearly linear, exception being made of the smaller measured diameters. If the lenses used were of poor quality and other conditions were such that it was not possible to obtain small star images such as are obtained at the present day, it is not surprising that equations of this type were obtained. In the

modern development of the subject, working along more correct lines, investigators have introduced logarithmic functions of time

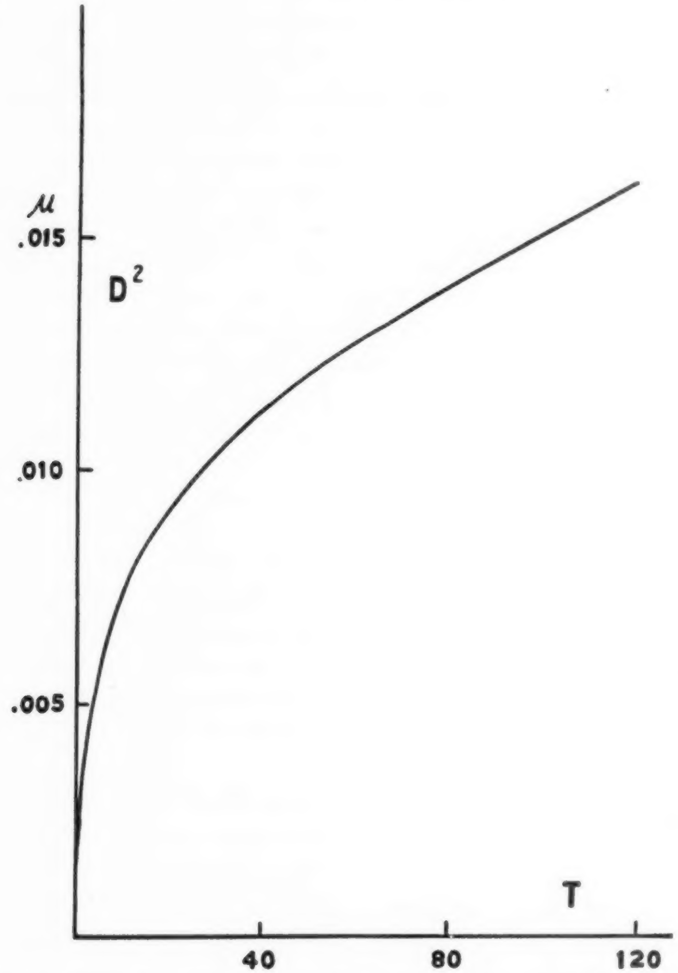


FIG. 3.—Comparison of Bond's formula for diameter of images with measures on modern dry plate.

and intensity in their equations, instead of those quantities themselves and their roots. This procedure is based on the principle that equal fractional or percentage variations in exposure produce

equal effects on the photographic plate, at least in its useful exposure region. Scheiner's formula,

$$d=a+b\log I, \tag{3}$$

is of this type. It has been shown to follow mathematically if the distribution of light at the edge of an image is of the simple form of Bouguer's law,

$$I=I_0^{-\kappa x}. \tag{4}$$

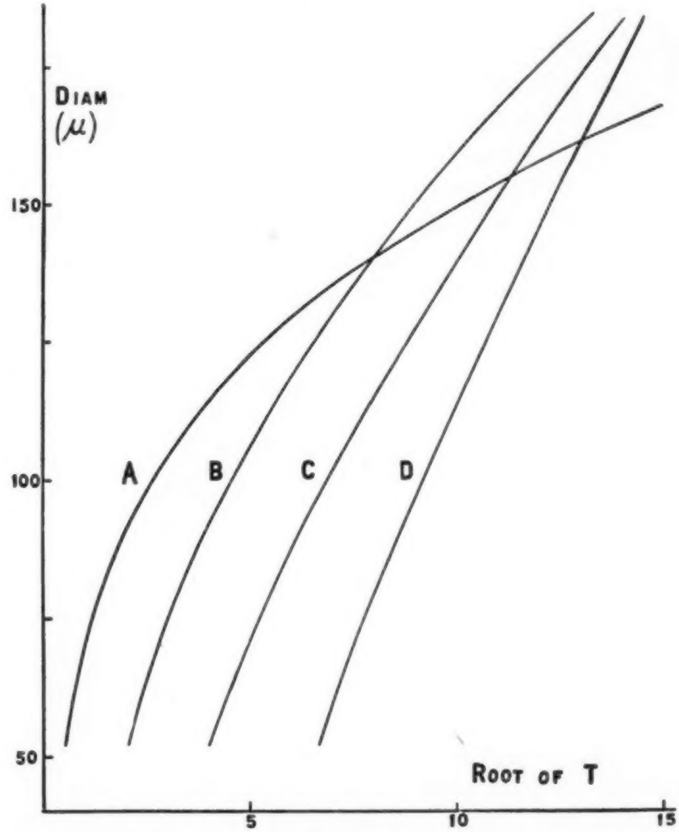


FIG. 4.—Diameters of images plotted against *A*, square; *B*, fourth; *C*, sixth; and *D*, eighth root of *t*.

This form of the intensity function has been adopted in deriving the series of equiluminous surfaces adjacent to the edge of an image (Fig. 1). It was soon found, however, that for the higher light-

intensities and longer exposures this simple formula no longer applied. It was superseded by the Greenwich formula which is equivalent to

$$\sqrt{d}=a+b \log I. \tag{5}$$

The relation between b and κ is

$$\kappa=\frac{2}{\text{mod.}} \cdot \frac{1}{b}.$$

This formula appears to satisfy the demands of most photometric work. So far as the writer is aware no question has been raised as to its applicability to very small star images such as are frequently obtained under the best conditions with well-corrected telescopes of not too great focal length.

In Table II the series of measured diameters adopted in the previous comparisons is used. The error of the Greenwich formula for diameters below 50 μ is seen to be very pronounced.

TABLE II

GREENWICH FORMULA COMPARED WITH OBSERVATION

Exposure Time	Measured Diameter	Diameter Greenwich Formula	Error of Formula
	μ	μ	μ
1ˢ0	19	30	+11
1.9	29	36	+ 7
3.8	39	43	+ 4
7.5	47	51	+ 4
15.0	57	58	+ 1
30.0	68	67	− 1
60.0	77	77	0
120.0	86	86	0
240.0	97	97	0
480.0	109	108	− 1
960.0	120	120	0
1920.0	130	131	+ 1

The diameters in Table II are plotted in Figure 5 according to the Scheiner equation (A) and the Greenwich equation (B). The Scheiner equation fits the observations, except in the case of the larger diameters. On the other hand the Greenwich equation is seen to be very seriously in error for the smaller diameters. It is not surprising that this error is not generally in evidence, since

the images which have been usually obtained in photometric work have not been sufficiently small, owing either to length of focus, poor optical corrections, or bad seeing.

The modification which is necessary in the Greenwich formula to make it fit the observations is quite obvious. It is only necessary to add a constant h to the measured diameter, the equation thus becoming

$$\sqrt{d+h}=a+b \log I\,. \tag{6}$$

Curves C, D, and E (Fig. 5) show the result for $h=50$, 100, and 150 respectively. In the last case, for $h=150$, the representation is

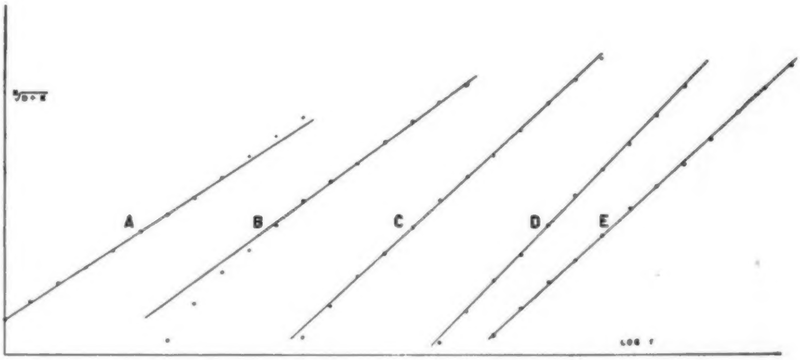


FIG. 5.—Diameters plotted against log t according to the Scheiner (A), Greenwich (B), and new formula (C, D, E).

entirely satisfactory. The correct value of h in any given case may be sufficiently well determined in a few minutes by graphical methods.

The problem of image growth will now be treated mathematically, in so far as this is practicable. It was formerly supposed that the growth of images was a chemical phenomenon, grains of silver bromide being supposed to develop by infection from neighboring grains. While there may be a certain measure of truth in this supposition, in so far as secondary actions are concerned, e.g., development of clumps of grains as a unit, it is not necessary to resort to hypotheses of this nature to explain the phenomenon, for the evidence shows that there is sufficient scattered light, in the region immediately surrounding an image, to account for its spread or

increase in size. It is quite impossible to deduce mathematically the curve of light distribution in a direct manner, on account of the number of separate factors entering, very few of which can be expressed in any kind of mathematical form. Attempts at solving the problem must therefore be restricted to the other end, working backward from photographic effect to light distribution, ignoring secondary development disturbances.

Let the light-intensity over the uniformly illuminated central stellar disk be denoted by I_0; the diameter of the disk by g, which depends on the optical system and upon the seeing, and in nowise upon the photographic plate. Let the light-intensity at a distance x from the edge of the uniformly illuminated central disk be denoted by I. Assuming Scheiner's equation of image growth, it has been shown that the light distribution is given by

$$I=I_0e^{-\kappa x}\,. \tag{7}$$

Assuming the Greenwich equation, the equation for light distribution can be shown to reduce to

$$I=I_0e^{-\kappa\left[\sqrt{x+g/2}-\sqrt{g/2}\right]}\,. \tag{8}$$

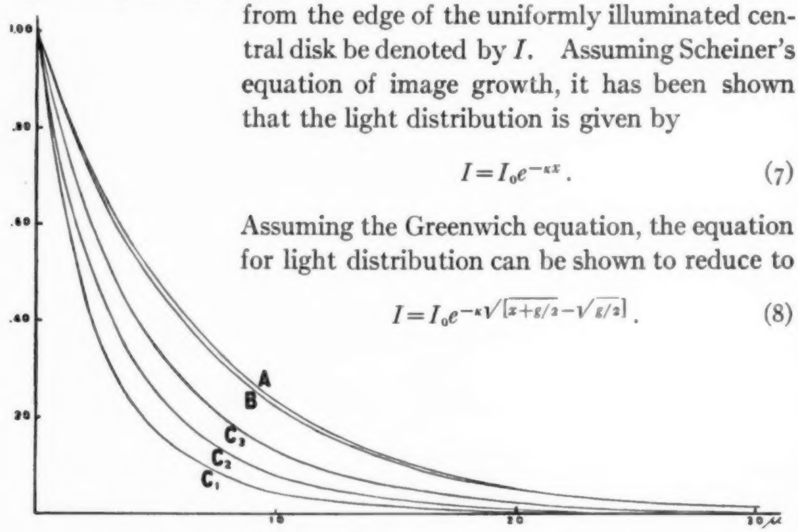


FIG. 6.—Light intensities near edge of an image: Scheiner formula, A; new formula, B; Greenwich formula, C_1, C_2, and C_3, for increasing values of g.

The new equation (6) leads to the distribution

$$I=I_0e^{-\kappa\left[\sqrt{x+\frac{g+h}{2}}-\sqrt{\frac{g+h}{2}}\right]}\,. \tag{9}$$

The curves of light distribution for the three cases of the Scheiner, Greenwich, and revised Greenwich formulae, computed from the foregoing equations, are given in Figures 6 and 7, using constants determined from Figure 5. Figure 6 is for the region

within 30 μ of the edge of the image; Figure 7 for the region 50 μ to 100 μ. The curves show the characteristic differences known to exist. For example, the Scheiner and the new formula agree near the edge of the image, as was to be expected, but diverge widely at greater distances. The Greenwich formula appears to stand alone at all distances, but in the region of greater distances this divergence is only apparent, for if in Figure 8 the ordinates of the Greenwich formula are multiplied by 5.9 the two curves are made to coincide.

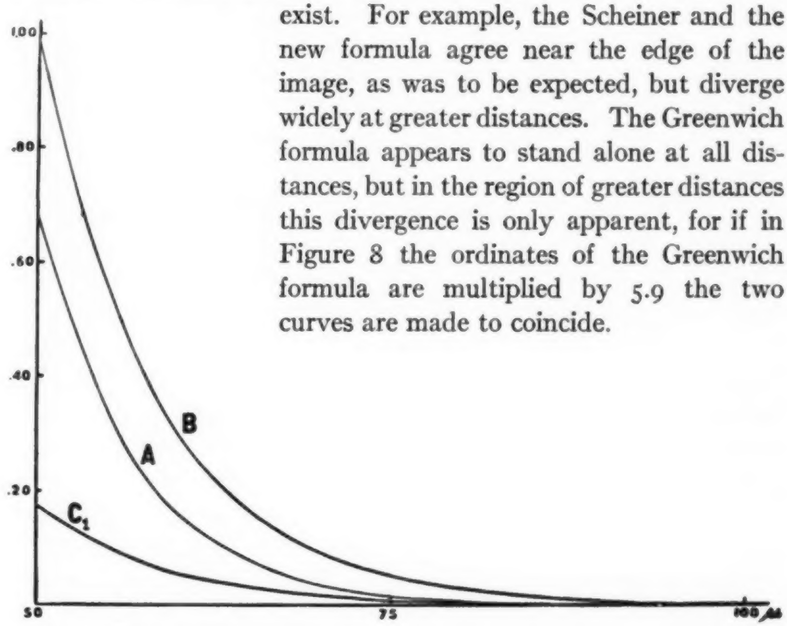


FIG. 7.—Relative light intensities, 50–100 μ from edge of image. Continuation of Figure 6.

Cause of the deviation from the Scheiner equation.—Some experiments have been made with the object of throwing light on the increase in the rate of growth of images with increased exposure. The data obtained by Bloch and Renwick (p. 348) on the opacity of emulsions are suggestive and useful in this connection. Using their equation for white light,

$$D=0.66\,W^{0.64}\,,$$

where D is the optical density of a film and W the mass of silver bromide. Calling x the distance below the surface,

$$W=a\cdot x,$$

where a is a constant. Also

$$D=\log\frac{I_0}{I}\,,$$

whence

$$I = I_0 e^{-\kappa x^{0.64}}. \tag{10}$$

This equation, which purports to give the distribution of light downward in an emulsion, is quite similar to the sideways light distribution (eq. 8) which was derived from the Greenwich formula, the only essential difference being in the exponent of x, which is 0.64 instead of 0.50.

Consider two round images of equal intensity on the photographic plate, with diameters respectively of the order of .05 mm and 1 mm, for example. If the physical constants of the emulsion are known, the light distribution is mathematically determinable, but, as pointed out above, the solution has never been attempted. Nevertheless, it is easy to see in a general way that the light distribution should not be the same for the two images, an equality which would be required by Bouguer's law. The falling off in intensity in the case of the larger image should be less rapid, on account of reinforcing illumination filtering out from the central region of the image. This reinforcement must be less effective in the case of the smaller image. The same reasoning applies to a certain extent in the case of two true star images formed by stars very unequal in magnitude, and is materially strengthened when expansion of the disks caused by atmospheric unsteadiness is taken into account. Accordingly, with increase in intensity, an increase in the rate of growth, such as is tacitly assumed in the Greenwich and the new equation, is to be expected. But the fact that the Greenwich equation is greatly in error for the small diameters must be remembered. It should be noted that Bloch and Renwick's equation (p. 348) was not based on data from extremely thin emulsions, so there is no experimental evidence that it actually holds for the top layers of an emulsion. Its applicability as embodied in the Greenwich equation, to the limited region immediately bordering the true edge of an image, is accordingly open to question.

Figure 8 shows the intensity of light distribution downward in an emulsion, computed from Bloch and Renwick's equation. W is taken as unity, which is a close approximation, and the thickness of the emulsion is taken equal to 20 μ. The distribution curve for

Bouguer's law is also shown for comparison. Assuming each layer of grains equal to 2 μ in depth, it is seen that the absorption in the first layer of grains at the surface is twice as much from Bloch and Renwick's equation as it is from Bouguer's. The same steep drop in vertical transmission is shown by Bloch and Renwick's curves as is shown by the sideways transmission curve computed from the Greenwich equation (Fig. 6). Reasoning from analogy, it is probable that in the first few layers of an emulsion the transmission downward is more nearly in accord with Bouguer than with Bloch and Renwick.

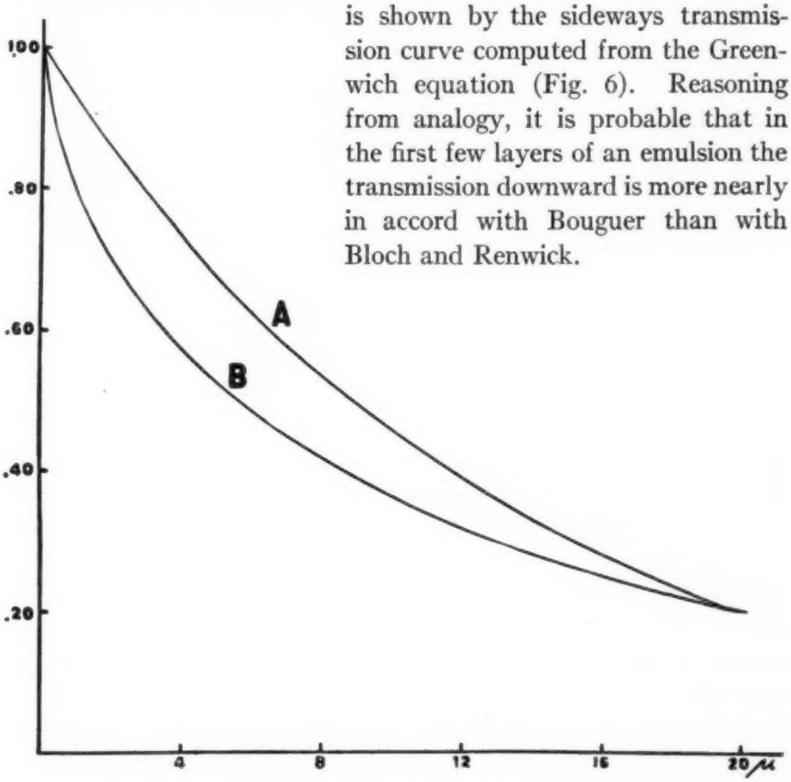


FIG. 8.—Light intensity at various depths in an emulsion according to Bouguer's (A) and Bloch and Renwick's (B) formulae.

To obtain data with respect to the dependence of Δ, the rate of growth, on the size of the unexpanded or optical image, a series of measurements was made, the results of which are contained in Table III. A test object having a series of nine circular apertures varying in diameter from 0.16 mm to 2.3 mm was photographed in the precision reducing camera, exposure times ranging from

1 second to 32 minutes, from which Δ was computed for each aperture. The plate used was Seed 23, quality of light, white.

In the case of the images of the five smallest apertures, no departure from Scheiner's law was apparent over an exposure range of 1 second to 32 minutes, or 1 to 1920. For the largest aperture, No. 9, the strictly linear relation (Scheiner's) held over a range of 1 second to 8 minutes or 1 to 480. Using the same test object and reducing the intensity of the light fifty times diminished Δ about 10 per cent for the larger holes, but had no effect on Δ for the smaller apertures. In addition it was found that for this case of diminished light-intensity there was neither increase nor decrease of the range over which the linear relation holds.

TABLE III

VARIATION OF Δ WITH SIZE OF IMAGE

Aperture	Diameter of Image (Geometrical)	Diameter of Image (Lightest Exposure)	Δ
	mm	mm	μ
1	.004	.010	9.0
2	.006	.012	9.2
3	.008	.015	9.5
4	.011	.018	9.7
5	.014	.020	9.9
6	.017	.031	10.8
7	.025	.046	11.2
8	.034	.058	11.5
9	.057	.085	11.9

In obtaining turbidity values for a very intense diffraction image, the diffusing screen between the light source and the test object was removed, and the light adjusted in position until it was in line with aperture No. 1 and the center of the lens. Under these conditions an exposure of 1 second gave an image of diameter 64 μ, which is as large as that previously obtained with a 6-minute exposure through the same aperture. From this it can be calculated that the increase in intensity is 300 times (6.2 magnitudes). The value of Δ was found to have increased from 9.0 to 12.5, which is approximately the value for aperture No. 9. However, the shape of the curve was quite different, the linear relation between diameter and exposure time holding only over the very limited range from 1^s to 30^s. Since in this experiment there has been no change in the

size of the geometrical image, nor in the character of the aberration or diffraction pattern, the conditions are analogous to the practical astronomical case of photographing a field of stars of varying brightnesses. The conclusion is that under these circumstances the *deduced relative magnitudes are dependent on the exposure time,* Δ being a function of the star's magnitude. This phenomenon is similar to the Purkinje or color-gradation change. In accurate astronomical photometry it is not to be overlooked.

The dependence of Δ upon telescope and plate is shown in the three paragraphs below which were communicated by J. A. Parkhurst, of the Yerkes Observatory, from a thesis of Miss Alice Farnsworth.

Forty-inch refractor.—The focal length is 62.5 feet. Size of minimum image, approximately .080 mm. There does not appear to be any straight-line portion to the diameter-magnitude curve. The following are the values of Δ, the plate being Cramer Iso.

Diameter mm	Δ mm
.080	.020
.500	.050

Δ having been defined as the increase in diameter for doubling of the exposure time, if the value is desired for unit increase in stellar magnitude, the values given must be multiplied by the ratio of 0.4 to log 2.

Twelve-inch aperture of 2-foot reflector.—The focal length is 93 inches. Size of minimum image, .017 mm. In the case of this telescope the relation between diameter and magnitude, as well as between diameter and log exposure time appear to be linear over the unusually wide range of approximately 8 magnitudes. The deduced values of Δ are:

Plate	Δ mm
Seed 30	.011
Cramer Isochromatic	.020

Ultra-violet 6-inch camera.—The focal length is 32 inches. Size of minimum image, .014 mm. The characteristics are the same as those of the reflector noted above. The values of Δ are:

Plate	Δ mm
Seed 30	.013
Cramer Isochromatic	.020

Probable errors.—The accuracy with which stellar magnitudes can be calculated by the turbidity of "growth of image" method depends upon two factors: (*a*) accuracy with which the diameters of the star disks can be measured, (*b*) magnitude of Δ. These are not independent, however, since the accuracy of measurement depends on the sharpness, which is a function of the turbidity. Calling the diameter d, and the magnitude m,

$$d=a+\frac{\Delta}{\log 2}\log I,$$

$$m=b+2.5\log I,$$

giving the increment or probable error relation

$$\delta m=\frac{0.75}{\Delta}\delta d. \tag{11}$$

For very sharp images the probable error δd is approximately $\pm 1\ \mu$. Taking Δ equal to $10\ \mu$, this relation gives $\delta m=\pm .075$ mag. Since the irregularities of the photographic emulsion are known to have an inherent probable error of approximately $\pm .06$ mag., the total probable error in this case becomes $\pm .096$. For larger values of Δ, the images become less sharp, so that δd increases, but not in the same ratio. For example, taking $\Delta=50\ \mu$, we should expect d to be approximately $\pm 2\ \mu$, giving $\delta m=.030$ mag. In this case the total error is $\pm .067$ mag. The advantage of greater turbidity which is gained both by proper choice of telescope and of emulsion is apparent.

The densitometric method, or measurement of density D for out-of-focus images, leads to a greater accuracy than can be attained by the turbidity method. In this case

$$D=a+\gamma\log I,$$

giving

$$\delta m=\frac{2.5}{\gamma}\delta D.$$

The error in magnitude thus is inversely proportional to γ or the contrast of the emulsion. If the exposures are properly made,

$\gamma = 1.5\pm$ for a fast plate, δD will vary but will not differ greatly from .01. Accordingly from the formula $\delta m = .017$ mag. In extreme cases, where contrasty plate and developer is used, $\gamma = 3.5$, giving $\delta m = .007$ mag. The inherent plate errors must of course be added to these values. According to recent tests by H. T. Stetson[1] a means has been found of reducing the plate error to one-half of its former amount. This will increase the value of the densitometric and the Stetson methods in stellar photometry relatively to the turbidity method.

Turbidity of fine-grain plates.—Table IV gives turbidity values of Seed lantern and yellow-dyed Seed lantern plates. The

TABLE IV

TURBIDITY

Aperture	Geometrical Diameter of Image	Seed Lantern		Yellow-Dyed Seed Lantern	
		Threshold Diameter of Image	Δ	Threshold Diameter of Image	Δ
	mm	mm	μ	mm	μ
1	.004	.007	8.0	.005	....
2	.006	.007	8.2	.005	....
3	.008	.007	8.5	.006	....
4	.011	.009	9.0	.008	....
5	.014	.011	10.4	.010	....
6	.017	.018	11.1	.014	5.1
7	.025	.021	11.3	.020	6.5
8	.034	.030	11.5	.025	6.9
9	.057	.056	11.6	.053	8.2

same test object as before was used. A considerable decrease in turbidity for yellow-dyed plates is apparent. The unexpected result is obtained that there is no difference between the turbidity of Seed 23 (Table III) and of Seed lantern.

Relative turbidity of star images and spectral lines.—In order to compare turbidities in this case, it is evident from the preceding that the images must be of approximately the same width. A test object was made having a slit aperture 1 mm in width with circular holes of the same width arranged on each side of the slit. This was

[1] *Popular Astronomy*, **29**, 287, 1921.

photographed in the reducing camera in the usual way on a Seed 23 plate. The results are as follows:

	Star Images	Slit
Width of image (1 sec. exposure)....	.082 mm	.068 mm
Observed Δ (exp. range 1^s to 120^s)..	8.2 μ	8.2 μ

A violet combined with an aesculine filter was used, which accounts for the small values of Δ compared with those in Table III, in which unscreened white tungsten light was used. The values of Δ for star images and slit are seen to be the same. This is contrary to expectation, for it was supposed that the slit would show greater turbidity, on account of the reinforcing illumination from a greater extent of illuminated area, just as it was found that larger star images, for this same reason, showed greater turbidity than small images.

Variation of Δ with wave-length.—The values of Δ for a number of plates differing in type were determined in the precision camera behind a series of color filters. Typical curves have been given in a former publication.[1] Δ has been found to be smallest in the violet, agreeing with theory. Curves of these types are of importance with respect to the photographic Purkinje phenomenon considered in the following section.

The extent to which the measured values of Δ depends on the optical system and on its accuracy of focus has not yet been determined. In the case of printing by contact (of a specially prepared metallic slit) with the emulsion, it can be taken for granted that the "optical" turbidity is negligible, so that the measured turbidity should be that of the emulsion itself. By comparing values thus obtained with those obtained in conjunction with optical instruments, the relative importance of optical and emulsion turbidities may be judged. Determinations of Δ for contact slit images were made for various wave-lengths on Seed 23 and Seed panchromatic plates. The printing light was derived from a monochromatic illuminator. The Δ-wave-length curves resulting are shown in

[1] F. E. Ross, "Photographic Photometry and the Purkinje Effect," *Astrophysical Journal*, **52**, 90, 1920.

Figure 9. Comparing with the values in Table III, there is seen to be substantial agreement. The conclusion is that for well-corrected optical systems the optical turbidity is negligible. The extremely small turbidity in the violet ($\Delta = 5\ \mu$) is noteworthy. It must not be overlooked that this is of profound importance in the

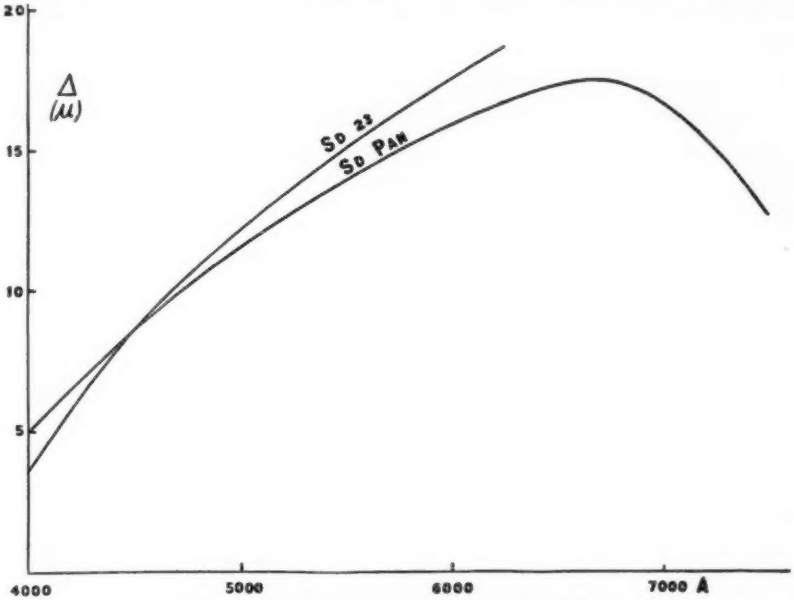


FIG. 9.—Turbidity depending on wave-length

Purkinje effect, whether considered from the standpoint of the turbidity or densitometric method of photometry.

PHOTOGRAPHIC PURKINJE EFFECT

A brief historical account will be given of the photographic Purkinje phenomenon, the change of gradation or contrast with change of color. That there is an appreciable effect of this kind appears to have been discovered nearly simultaneously by Sir W. Abney[1] and J. Precht[2] in 1899. Precht discovered the phenomenon

[1] "On the Variation in Gradation of a Developed Photographic Image When Impressed by Monochromatic Light of Different Wave-Lengths," *Proceedings of the Royal Society*, **61**, 300, 1901.

[2] "Photographisches Analog zu Phanomen von Purkinje," *Arch. wiss. Photographie*, **1**, 277, 1899.

through comparing densitometric results obtained by using as sources of illumination a benzene lamp and an amyl-acetate lamp, respectively poor and rich in actinic rays, it is claimed. He concluded: (1) "Die relative chemische Helligkeit zweier Lichtquellen ist abhängig vom absoluten Intensität." (2) "Die relative chemische Helligkeit zweier Lichtquellen ist abhängig vom absoluten Wert der Expositionszeit."

C. Jones[1] made a careful study of the phenomena at about the same period. He concluded that in general the gradation increases continuously from the ultra-violet to the red. An exception is found when development is in pyro-ammonia, in which case the gradation curve was found to be reversed.

The following quotation from Abney clearly summarizes his conclusions:

Nearly two years ago, in an article in *Photography* I indicated that a variation in gradation due to differences in the monochromatic light in which the exposure was made did exist, and some six months ago Mr. Chapman Jones, in a paper communicated to the Royal Photographic Society, independently announced the same result from experiments made principally with orthochromatic plates with light passing through various colored media and he generalized from his experiments that the smaller the wave-length the less steep was the gradation, the ultra-violet rays giving the least steep and the red the most steep gradation. My experiments which at that time had been partially completed did not bear out this generalization to the full when pure silver salts were used, and my subsequent measurements with them show that the least steep gradation is that given by the monochromatic light to which the simple silver salt experimented with is most sensitive, and that the gradation becomes steeper as the wave-length of light employed departs in either direction in the spectrum from this point, the steepest gradation being given by the extreme red. The case of orthochromatic plates in which is a complex mixture of silver salt and dye is necessarily less simple, involving consideration of the localities in the spectrum to which the dye or dyes, together with that of the silver salt, are most sensitive. For this reason the simple salts have been experimented with in preference to the more complicated organic compounds. It may be thought that these results might be peculiar to the salt of silver experimented with. A further series of experiments were conducted with the chloride of silver in gelatine. The maximum sensitiveness of these plates was found to be near H ($\mu = 3968.6$ A) in the solar spectrum. The gradation was found to be least at this point and increased when rays on each side of this point were

[1] "The Effect of Wave-Length on Gradation," *Photographic Journal*, **24**, 279, 1900.

employed to act on the film. In the blue, near the F line, where the sensitiveness of the plate was very small, the gradation was excessively steep, as it also was in the extreme ultra-violet.

Abney's conclusion, that the least gradation is at the point of maximum sensitivity, is a beautiful and somewhat startling theorem. Abney himself appears at a loss to account for it. "What scientific explanation there is of this difference in true gradation factor is hard to say." The following explanation is proposed by the writer. It has been shown in a previous paper[1] by the writer that for emulsions composed of grains of equal sensitivity the gradation or gamma is a maximum, and that as the number of groups of grains of varying sensitivity increases, gamma decreases. A simple case in point is the mixture on one plate of a fast and a slow emulsion. It appears quite probable that in the case of the emulsions studied by Abney, a greater number of groups, with no inert grains, come into action at the wave-length where the general sensitivity is a maximum. This should result in a lower gamma, as just shown.

Contrary to the results quoted so far, G. Leimbach[2] obtains no change of gradation with wave-length. "In Bereiche der normalen Belichtung wachst die Schwarzung mit zunehmender Belichtungszeit nach demselben Gesetz, "or," Die Gradation ist für alle Wellenlangen die selbe."

The astronomical importance of the subject is well illustrated in the attempts which have been made to prove from photographic data that light suffers an appreciable absorption in its passage through space, which were based upon the assumption that the photographic gradation is the same for all colors. That the data do not allow such conclusions to be drawn has been shown by J. A. Parkhurst[3] and by H. E. Ives.[4] From his own investigation of the subject Parkhurst concludes ". . . . Experiment with the photographic color-filter furnishes no evidence for or against an

[1] *Loc. cit.*, p. 265.

[2] "Die Absolute Strahlungsempfindlichkeit von Bromsilberplatten gegen Licht verschiedener Wellenlange," *Zeitschrift für wissenschaftliche Photographie*, **7**, 181,1901.

[3] "The Evidence from Photographic Color Filters in Regard to the Absorption of Light in Space," *Astrophysical Journal*, **30**, 331, 1909.

[4] "Some Photographic Phenomenon Bearing upon Dispersion of Light in Space," *ibid.*, **31**, 157, 1910.

absorption or scattering of light in space." Ives shows that a conclusion of selective space-absorption of light is based upon the assumption that the gradation of the photographic plate is the same for all colors. Ives's paper on the subject is of importance in indicating causes underlying the phenomenon, and in giving reasons for the contradictions found by different investigators. He concludes that the factors of importance in determining the variation of gradation with wave-length are (1) penetration of the dye in the film; (2) penetration of the light; (3) penetration of the developer. He notes the important fact that when plates are color sensitized by bathing, the penetration of the dye into the emulsion is superficial. In the case of plates color sensitized by the manufacturer, the dye is incorporated in the emulsion before coating, so that the light-sensitive layer is much thicker and consequently a much higher gradation is obtained. "With a Cramer instantaneous isochromatic plate, the red gradation was steeper than the blue. With a Seed 26, bathed for two minutes in a 1:100,000 solution of pinacyanol, the blue gradation was steeper than the red." Ives suggests that Leimbach's negative result was due to surface development, since he developed in ferrous oxalate developer for a short time only.

J. A. Parkhurst[1] has sought to determine directly the change of gradation of the photographic plate for stars of different classes or colors. He finds a difference of 1.5 per cent in the direction of higher gradation for the red stars, but concludes: "We, therefore, reach the conclusion that for Seed 27 plates and for extra-focal images the slight systematic effect which may be present is nearly or quite masked by the accidental errors arising principally from the lack of uniformity in the film."

No attempt has been made in the foregoing review to enumerate all the investigations bearing upon this very important subject. The contradictions and confusion are evident without further citation. Careful study of the methods employed by the various investigators does not in general disclose any lack of care in details or narrowness in treatment. Without any pretense to being

[1] "A Property of the Photographic Plate Analogous to the Purkinje Effect," *Astrophysical Journal*, **49**, 202, 1919.

exhaustive, many of the investigations were admirably planned and carefully carried out. Assuming as we must therefore that no flagrant errors were made by any or by all of the investigators, the only possibility of accounting for the discrepancies appears to be to assert that the wave-length effect upon gradation is strongly variant with the particular emulsion, and to a possibly lesser extent with developer and development conditions. It cannot be said that this dependence was not present in the minds of many of the investigators for, in several cases, a large number of plates of various kinds, including blue sensitive, isochromatic and panchromatic were used. Also more than one developer was in general employed.

H. E. Ives (p. 370) has pointed out why no general statement of the gradation wave-length effect can be made, and enumerated some of the important causes of variation in results obtained. Besides those mentioned by him, there are doubtless others. There is, for instance, the Eberhard effect, or retardation of development dependent upon the *density* of an image and upon its *size*. Since astronomers are concerned in general only with very minute images, it is quite clear that laws of photographic action worked out in the laboratory where much larger density areas are in general employed, are not entirely applicable.

The tendency of a photographic plate to reversal, a phenomenon which, according to the views of some investigators, begins at the first moment of exposure and gains strength with its advance, must have an important bearing upon the gradation of the plate. Abney has shown that pre-exposure of a plate to red light accelerates the reversal tendency. It is quite clear from this therefore that the composition of the incident light must have an indirect or catalytic action on the plate, aside from its direct latent-image forming characteristics. Accordingly, the *quality* of the light used by the experimenter has an indirect effect on gradation and can account for some of the differences obtained. That the reversal action is of importance in practical photography is shown by the experiments of Channon[1] on exposures of great length.

Consideration of the action of reducers gives a helpful point of view. It is well known that the action of some reducers is con-

[1] "Studies in Photographic Science," *Photographic Journal*, **50**, 164.

fined to the surface of the image, while others appear to act uniformly through the depth of the image. This differential action depends upon relative rates of diffusion of reactants and reaction products compared with the velocity of the chemical actions. Similar principles can be conceived to hold in ordinary development. In fact the Eberhard effect is accounted for chemically in this way. Accordingly rates of diffusion through gelatine, with the many chemical factors which control and change it, become important factors in depth development, which has been shown to play an important part in gradation phenomena.

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THE SPARK SPECTRUM OF GALLIUM IN AIR AND IN HYDROGEN

By ELIAS KLEIN

ABSTRACT

Spark spectrum of gallium, 2177 to 6414 A, in air and hydrogen, was photographed with the aid of a Littrow type spectrograph, and the wave-lengths of 83 gallium lines, 14 of them questionable, are given accurate to about ±0.05 A. Of these 23 had been measured by Uhler and Tanch and 26 by Saunders, but 48 are new. The spark in hydrogen was tried because the spark in air oxidizes the gallium and then the spectrum becomes less rich in lines. To eliminate lines due to possible impurities, spark spectra of Cu, Fe, Pb, In, Zn, and Sn were photographed next to the gallium spectrum.

APPARATUS AND METHODS

The spectral regions studied were photographed with a large quartz spectrograph, of the Littrow type, made by the Adam Hilger Company, Ltd., London. For this instrument the spectrum range between λ 2000 and λ 8000 is divided into three portions which are approximately: λλ 2000–2500; λλ 2500–3400; λλ 3400–8000. The dispersion for the middle of each of the above regions is roughly, 2 A per mm; 6 A per mm; 15 A per mm.

The gallium used for the spark in air was prepared and purified by Professor Uhler according to the method of Browning and Uhler.[1] A bead of this metal, about 3 mm in diameter, was placed in a small cavity made in the end of a copper rod 10 mm in diameter. This served as the lower electrode. The upper electrode was a No. 5 copper wire filed to a point. For the spark in hydrogen (discussed later) the gallium was electrolytically deposited from a decidedly alkaline solution of gallium chloride. The original solution contained indium sulphate, gallium chloride, and ammonium formate, the indium having been removed first by electrolysis. The gallium thus obtained was placed in a quartz tube which was evacuated and heated with a blast lamp to the softening point of the quartz. The pumps were kept in operation during this process. Owing to the very low vapor pressure of gallium most of the metals (Zn, Pb, Sn, In) which occur as impurities in it are distilled on the cooler portions of the tube (and the trap joining it) at a much lower tempera-

[1] Uhler and Browning, *American Journal of Science*, **41**, 351; **42**, 397, 1916.

ture than the boiling point of gallium. Thus the gallium is purified and remains in liquid form at the bottom of the tube.

The spark in air as well as that in hydrogen was produced by a 500 watt transformer (powerfactor 1/3) rated at 15,000 volts and made by the American Transformer Company. The primary current, 110 volts 60 cycles, varied from six to fifteen amperes, depending upon the metals used in the spark gap. In parallel with the spark gap, which was about 5 mm, was connected a Leyden jar battery of capacity approximately .005 μf. The current in the oscillatory circuit, as measured by a hot wire ammeter, was over 10 amperes. The period of this circuit (gap in air) was about 10^{-6} seconds.

The time of exposure for the spark varied from 5 to 20 minutes in air, and from 20 minutes to 2 hours in hydrogen. Slit widths of 0.02–0.04 mm were used throughout this work.

Generally Seed plates Nos. 26 and 30 were used and found quite satisfactory. For the region between λ 6000 and λ 8000 special plates were obtained from the Eastman Kodak Company, through the courtesy of Dr. C. E. K. Mees. These plates served their purpose admirably. A simple hydrochinone developer according to L. E. Jewell[1] was used throughout.

The spectra of copper and iron were used for standard comparison. The most likely impurities (other than copper and iron) in the metal used for the present investigation are lead, indium, zinc, and tin. The spectra of these metals were photographed above and below the gallium spectrum in each case, without displacing the plate. This method of eliminating foreign lines saved much time in the final measurements of the plates. All the plates were measured with a Gaertner engine which had been modified and tested by Professor Uhler.

SPARK IN HYDROGEN

It was originally intended to get the spark spectrum of gallium in air only, and several plates were taken for each region, thus covering the entire range of the spectrograph. Upon studying the plates in detail it was observed that, in general, in any given region

[1] L. E. Jewell, *Astrophysical Journal*, 11, 242, 1900.

of the spectrum the same sample of gallium radiated a decreasing number of lines with successive exposures of identical duration. This was due to the fact that the gallium oxidized very readily but superficially when heated, and that the oxide of gallium (Ga_2O_3) apparently radiated only the strongest lines of the metal. This difficulty could have been overcome by continually removing the outer oxide layer, thus bringing the metallic surface into sparking communication with the upper electrode. This method, however, was considered impracticable because of the great scarcity of the metal, and the long, troublesome process of again reducing the oxide of gallium to the metallic form. Besides, one is never certain, by this method, that he has got all the lines gallium is capable of radiating under the given sparking conditions, although it later appeared that some of the exposures in air showed as many lines as those in hydrogen. It was necessary, therefore, to seek an oxygen-free medium for the gallium spark. Nitrogen proved undesirable, due to its many bands which masked the fainter gallium lines. Helium, as supplied by the Navy Department, was found to contain too much nitrogen to be useful in this investigation. Commercial electrolytic hydrogen, as supplied by the Tariffville Oxygen Company, Connecticut, proved quite pure and gave satisfactory results.

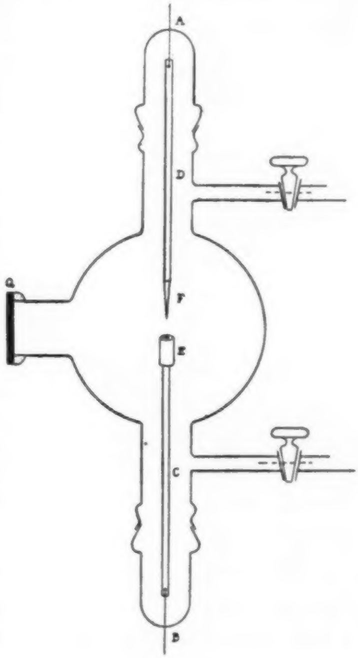


FIG. 1

The spark chamber employed was as shown in the accompanying sketch. It consisted of a glass bulb about six inches in diameter. Into the ground-joint tubes were sealed platinum wires, *A*, *B*. To the inner ends of these were joined aluminum rods, *B*, *C*. *E* is a short copper rod having a cavity in its upper end where the gallium

was placed. *F* is a No. 5 copper wire. The quartz plate, *Q*, was cemented to the bulb with sealing wax.

The chamber and system of drying bulbs to which it was attached were evacuated to a pressure of about 10^{-5} cm of mercury by means of a Western Electric diffusion pump, and a Gaede rotary pump. The apparatus was then filled with dried hydrogen to atmospheric pressure and was cut off from the supply tank by means of a stop-cock. During each exposure a fan cooled the bulb to keep the pressure constant and the sealing wax about the quartz window from melting.

DETERMINATION OF WAVE-LENGTHS

The wave-lengths of gallium were computed by means of the simplified Hartmann dispersion formula,

$$\lambda=\lambda_0+\frac{C}{N_0-N}\,.$$

The constants λ_0, C and N_0 were calculated from the superimposed copper or iron spectrum. The interval for which these constants were determined never exceeded 200 A units. In most cases 50 or 100 A were used. Each set of plates, those taken in hydrogen as well as those taken in air, were measured, and independent computations of wave-lengths were made for each.

The wave-lengths given in the following table are the averages of the two sets of computations. Faint lines, the wave-lengths of which agreed with those of more intense lines in the spectrum of a common element, were considered as impurities. Questionable lines are indicated as such, and should be interpreted as faint lines whose wave-lengths do not agree with suspected impurity lines by 0.05 A or 0.10 A. The limit of accuracy of the wave-lengths shown in the table is approximately 0.05 A.

Column two contains a list of wave-lengths of the gallium spark spectrum which Professor F. A. Saunders had occasion to measure recently and which he very generously turned over to the author for comparison. A Hilger Quartz Spectrograph Type "C" was used for photographing these lines. The gallium contained traces of copper, chromium, and manganese.

TABLE I

TABLE OF WAVE-LENGTHS IN INTERNATIONAL ANGSTROMS

No.	Uhler and Tanch Arc in Air	Saunders Spark	Present Work Spark	Estimated Intensity	Remarks
1	2171.9	....	....	......	
2	....	....	2176.76	3	broad, hazy
3	2195.665	....	2195.63	2	H_2 plates only
4	....	....	2205.91	2	hazy
5	2211.753	....	....	......	
6	2218.039	....	....	......	
7	2236.103	....	2236.17	3	fairly sharp
8	2255.034	2255.60*	2255.29	2	hazy
9	2259.227	........*	2259.24	1	faint
10	....	2266.86*	2266.84	2	diffuse
11	....	........*	2285.19	2	hazy H_2 plates only
12	2294.202	2294.17*	2294.19	2	diffuse
13	2297.869	2297.90*	2297.89	2	fair
14	2338.293	2338.25	2338.24	3	broad
15	2338.596	....	....	......	
16	....	....	2359.48(?)	1	H_2 plates only
17	2371.325	2371.30	2371.29	3	sharp
18	2418.699	2418.72	2418.69	4	sharp
19	....	....	2422.61	3	broad, diffuse
20	....	....	2423.14	2	hazy, diffuse
21	....	2424.27	2424.29	3	fairly sharp
22	2450.078	2450.02	2450.04	6	diffuse
23	2500.187	2500.18	2500.17	7	diffuse
24	2500.714	2500.74	2500.72	3	diffuse
25	....	....	2588.92(?)	3	hazy H_2 plate only
26	....	....	2589.73(?)	2	hazy H_2 plate only
27	....	....	2602.04	2	very hazy
28	2659.873	2659.90	2659.89	7	fair
29	....	2700.55	2700.56	4	sharp
30	....	2702.39	2702.42(?)	1	faint on H_2 plate only
31	2719.664	2719.76	2719.59	8	fairly sharp
32	....	2780.22	2780.12	9	very broad
33	....	2830.24	2830.19	1	very faint
34	....	....	2836.87	1	very faint
35	2874.240	2874.35	2874.24	9	diffuse
36	....	2929.81	2877.50(?)	3	sharp
37	2943.639	2943.60†	2943.59	10	diffuse
38	2944.175	2944.27†	2944.18	9	diffuse
39	....	2987.97	2987.99	4	diffuse
40	....	2997.93	2997.99	3	diffuse
41	....	3004.14	3004.10	6	fairly sharp
42	....	....	3015.60	2	very hazy
43	....	3024.38	3024.33	1	faint
44	....	3058.13	3058.15	3	diffuse
45	....	....	3058.92	1	faint H_2 plate only
46	....	3064.88	3064.45	2	sharp
47	....	....	3072.14	3	fair
48	....	3074.22	3075.87	2	fair
49	....	....	3093.22(?)	2	sharp
50	....	....	3124.90	1	faint
51	....	....	3181.44	2	diffuse

TABLE I—*Continued*

No.	Uhler and Tanch Arc in Air	Saunders Spark	Present Work Spark	Estimated Intensity	Remarks
52			3189.01	1	faint
53			3190.46(?)	2	sharp
54			3192.86(?)	2	sharp
55			3375.32	7	diffuse
56			3417.12	5	hazy
57			3473.39	1	faint
58			3479.46	5	diffuse
59			3517.21	4	diffuse
60			3576.96	5	fairly sharp
61			3650.73(?)	1	faint hazy
62			3701.61	2	faint hazy
63			3730.03	6	diffuse may be doublet
64			3806.87	4	diffuse
65			3944.29	2	sharp
66	4032.975		4032.98	10	fair hazy edges
67			4069.52(?)	2	diffuse
68	4172.048		4172.05	10	fair
69			4250.67	3	diffuse
70			4254.13	2	hazy and diffuse
71			4256.17	2	hazy
72			4261.93	4	hazy
73			4616.43(?)	2	fairly sharp
74			4417.69(?)	2	fairly sharp
75			4820.59	2	sharp
76			4864.92	4	diffuse
77			4995.61	3	sharp
78			5338.88(?)	2	diffuse
79			5360.09	4	diffuse—looks like doublet
80			5844.22(?)	4	very diffuse
81			5992.34	3	hazy
82			6396.61	5	fair
83			6414.01	4	fair

* Iron standards are very weak here. Precision reduced.

† Not completely resolved. Might be a pair.

The agreement between columns two and three, although by no means perfect, is as close as could be expected when the following factors are taken into consideration. (1) The difficulty in setting on the lines because of the continuous background, especially on the plates taken with gallium in hydrogen which required long exposures; and, the easy reversals of the gallium lines. (2) The difference in dispersion between the two instruments used. Lines 47 and 48 have probably coalesced into one line on the plate of Professor Saunders. The mean position of these two lines is about 3074.00 which is no further away from 3074.22 than is line 46 from

the corresponding line of Professor Saunders. He designates 3074.22 as "very hazy."

The author has so far been unable to extend the gallium series beyond those shown by Paschen and Meissner,[1] and Uhler and Tanch.[2]

The author is indebted to Professor H. S. Uhler for suggesting the problem and for the interest he showed during the progress of this investigation.

SLOANE LABORATORY
YALE UNIVERSITY
August 1921

[1] *Annalen der Physik*, **43**, 1223, 1914.

[2] *Astrophysical Journal*, **55**, 291, 1922.

THE ABSENCE OF OXYGEN AND WATER-VAPOR LINES FROM THE SPECTRUM OF VENUS[1]

By CHARLES E. ST. JOHN and SETH B. NICHOLSON

ABSTRACT

Absence of oxygen and water-vapor lines from the spectrum of Venus.—It has been generally assumed that the atmosphere of Venus is like our own, containing both oxygen and water vapor, but the spectroscopic evidence for this assumption is very doubtful. Recently spectrograms have been made with the Snow telescope and the Littrow grating spectrograph at times when the relative velocity of Venus and the earth was sufficient to separate completely corresponding lines produced by the two atmospheres. Special attention was given to the lines of water vapor near λ 5900 and to the bands **a** and **B** due to oxygen. No trace of any line due to the planet's atmosphere was observed. It is estimated that in the path traversed by the solar light through Venus' atmosphere there must be less than the equivalent of one meter of oxygen, less than one-thousandth of that in our atmosphere, and less than one millimeter of precipitable water vapor.

Atmosphere of Venus.—Various observational facts and theories concerning this atmosphere *are discussed.* Although the new data prove that there is no appreciable amount of oxygen or water vapor above the visual surface of Venus, nothing is definitely known about the elevation of this surface above the presumably solid surface of the planet, or whether this reflecting layer is composed of cirro-strati, of haze, or of clouds of dust produced by violent atmospheric circulation.

It has been generally assumed in the literature on planetary atmospheres that the atmosphere of Venus is similar to our own; in particular, that oxygen and water vapor are present. Secchi,[2] 1868, noted in the spectrum of Venus a nebulous band on the red side of the D line and also one to the violet, the δ of Brewster. He observed these with Venus at a high altitude and when the air was so dry that he assumed that they were not of telluric origin. Huggins[3] had previously, 1863, studied the spectrum of Venus, Mars, Jupiter, and Saturn. Of Venus he says:

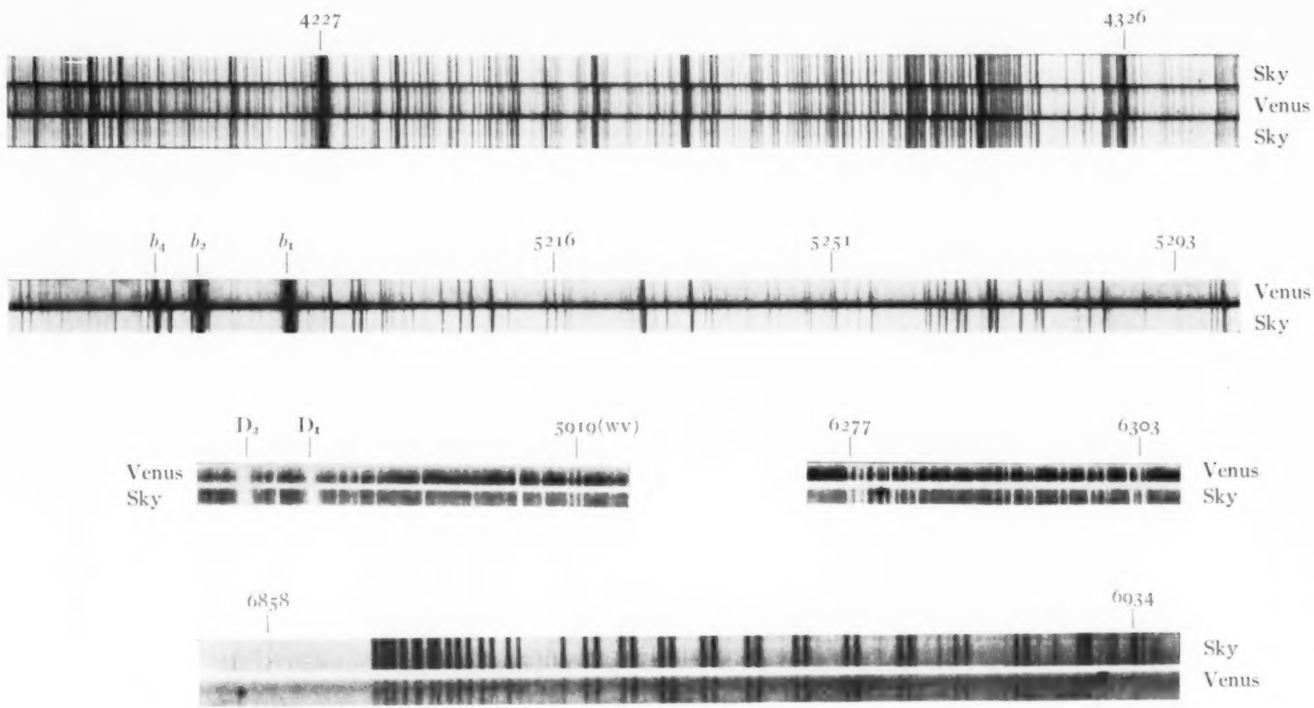
> The light of Venus gives a spectrum of great beauty. The line D was seen double, B, C, and numerous solar lines to a little beyond G, were distinctly visible; and the principal of these were measured and found to agree with corresponding lines in the solar spectrum. Lines other than these, and in the position in which the stronger atmospheric lines present themselves, were carefully looked for, but no satisfactory evidence of any such lines has been obtained.

[1] *Contributions from the Mount Wilson Observatory*, No. 249.

[2] *Spettrie Protuberanze*, p. 24, 1869.

[3] *Philosophical Transactions of the Royal Society*, **154**, 422, 1864.

PLATE XIII



SPECTRA OF VENUS AND SKY

The basis of the assumption permeating the literature on planetary atmospheres is found in the observations of Vogel.[1] Of his observations on the oxygen band at λ 6275 and the water-vapor bands at λλ 5945 and 5925 he says:

> Die Venus stand bei den Beobachtungen so hoch, dass man nicht annehmen kann, dass unser Atmosphäre einen merklichen Einfluss auf das Venusspectrum ausgeübt habe, die Linien oder Streifen sind daher entschieden dem Venusspectrum eigen und deuten auf die absorbirende Wirkung einer Atmosphäre.

Scheiner[2] in his review of the question sums up as follows:

> There seems to be no doubt that the telluric lines appear stronger in the less refrangible portions of the spectrum of the planet than in the solar spectrum. There can therefore be no doubt that the atmosphere of Venus exerts an absorption similar to that of our own, and hence the nature of the two atmospheres must be similar. . . . . We may safely assume that the clouds of Venus consist of condensed aqueous vapor, thus again resembling those of the earth.

Arrhenius[3] is still more specific and states that:

> The humidity (on Venus) is probably about six times the average of that on the earth or three times that in the Congo where the average temperature is 26° C. The atmosphere of Venus holds about as much water vapor 5 km *above* the surface as does the atmosphere of the earth *at* the surface. We must therefore conclude that everything on Venus is dripping wet. The vegetative processes are greatly accelerated by the high temperature. Therefore the life time of organisms is probably short.

The fallibility of these older visual observations, in which conclusions are drawn from estimates of the relative intensities of integrated groups of lines in the spectrum of Venus and of the moon or sky, is shown by comparing Vogel's observations in the green region with our spectrograms of Venus and the sky. Vogel notes, among others, groups at λ 5216 and λ 5251 as strengthened in the spectra of Venus, and Scheiner accepts the observations as definite proof of the similarity of the atmosphere of Venus and the earth. Arrhenius' conception of supertropical moisture and luxuriance of vegetation is founded likewise on these older observations. These groups are resolved on the Mount Wilson spectrogram, Plate XIII, and our

[1] *Untersuchungen über die Spectra der Planeten*, pp. 10–16, 1874.

[2] *Astronomical Spectroscopy*, pp. 197–198, 1894.

[3] *The Destinies of the Stars*, pp. 250–251, 1918.

examination shows no strengthened lines in the spectrum of Venus. The lines of both groups are identified as iron lines of intensity 2 and 3 on the Rowland scale. The degree of dependence that can be placed upon the Mount Wilson spectrograms may be inferred from observations at λ 5293 of some lines which are marked Awv? and 00 intensity in the Rowland table. These were clearly displaced in the spectrum of Venus, with no lines visible in the undisplaced positions. An examination of juxtaposed spectra of the east and west limbs of the sun confirms this evidence that the lines are entirely solar and not atmospheric.

These older visual observations depended on estimating the relative intensities of integrated groups of lines in the spectra of Venus and skylight. Lowell[1] suggested measuring the position of the oxygen and water-vapor lines to see whether they were affected by the Doppler shift, and V. M. Slipher obtained spectrograms of Mars and the moon and of Venus and the sky with a dispersion of 50 A per mm at 6100 A.[2] Of the spectrograms of Mars, Lowell says:

> The results at first seemed significant. To the writer's eye the shift of the solar lines under the microscope was perceptible though slight and in the shift the a band seemed to share. The lines of water vapor near D though present in both spectra were not strong enough to make much deduction possible.

Of Venus he says,

> Here again eye estimates by the writer subscribed to a shift in the a band, the water vapor lines, very faint, concurring; . . . . As regarded differences in density, none was perceptible between either the Martian and the lunar or the solar and Venusian, either in the oxygen a band or the water vapor lines near D. Water vapor is probably non-existent on the illuminated side of Venus and extremely scarce on Mars. As for oxygen the results above show that the spectroscopic method is hardly a delicate enough one in this respect to decide the question.

And Slipher remarks:

> Examination and measurement of them led to the same uncertain results as in the case of those of Mars. Although this attempt has failed to detect aqueous vapor in Mars and Venus, the conclusion should not be drawn that it

[1] *Lowell Observatory Bulletin*, No. 17, 1905.

[2] Campbell and Albrecht in observations on Mars quite independently applied the Doppler-Fizeau method to a planetary atmosphere. *Lick Observatory Bulletin*, **6**, 11 (No. 180), 1910.

does not exist in their atmospheres, nor that it will always remain impossible to discover it spectrographically.

This brief account reviews the spectroscopic evidence upon which the current views in respect to the constitution of the atmosphere of Venus were founded when we began our observations in March, 1921, and indicates the extreme detail reached in their interpretation. At the Berkeley meeting of the Astronomical Society of the Pacific in August, when we reported our results, Dr. Slipher also reported results of his recent observations in which he used relative intensity as a criterion, and obtained similar negative results.

INSTRUMENTAL METHODS

The spectrograms were obtained with the Snow telescope and the Littrow grating-spectrograph.[1] To Professor Wood we are greatly indebted for the loan of a large grating very bright in the first order. It was ruled by Anderson and has a surface of 95×170 mm. The dispersion in the first order of the 18-foot spectrograph is 3 A per millimeter. For the α band and the 5900 region the plates were Seed 23's, sensitized to red light by pinacyanol. Ilford Special Rapid panchromatic plates, hypersensitized with ammonia, were used for the B band, λ 6867.

The wave-lengths of eight water-vapor lines near D and of seven oxygen lines in the α band were measured on five spectrograms of Venus and on three of the sky, using I.A. wave-lengths of solar lines, corrected for motions, for standards of reference. On three of the spectrograms of Venus the wave-lengths of the water and oxygen lines were referred also to iron arc lines; the spectrum of the arc was put on intermittently during the long exposures on Venus. On a fourth spectrogram of the sky they were referred to the lines of a simultaneously exposed iron arc. The wave-lengths thus found were compared with the wave-lengths of the same telluric lines, obtained under much higher dispersion, given in Table I.

From the variation among the differences, Rowland *minus* International (R − I in Table I), the relatively low accuracy of measurement of water-vapor and oxygen lines is apparent, even on spectrograms with a scale of 0.72 A per millimeter. For the

[1] *Mt. Wilson Contr.*, No. 208; *Astrophysical Journal*, **54**, 381–382, 1921.

oxygen line λ 6278, which is double on the Mount Wilson plates, the mean wave-length of the two components is used for the comparison with the Rowland value. In such a series of measurements as is presented in this paper, where the scale is 3 A per millimeter, a precision in the final means greater than 0.005 A is hardly to be expected. The results of the measurements are shown in the eighth column of Table II under Δλ, the Δλ being the difference between the observed wave-lengths and those of Table I.

TABLE I

WAVE-LENGTHS OF THE COMPARISON LINES

Water Vapor* I.A.	R−I	Oxygen† I.A.	R−I
5885.982	+0.211	6278.106	+0.197
87.227	0.218	81.186	0.201
87.665	0.215	81.963	0.201
5909.001	0.212	92.171	0.202
19.058	0.218	92.967	0.203
19.646	0.214	95.186	0.203
24.276	0.214	95.968	+0.202
32.097	+0.219		

* *Mt. Wilson Contr.*, No. 223; *Astrophysical Journal*, **55**, 36, 1922.

† Unpublished results by St. John and Babcock.

Because of the dearth of reference lines, a different method of measurement was adopted for the oxygen lines in the B band, λ 6867. The positions of nineteen oxygen lines in the spectra of Venus were compared with their positions in the spectra of the sky taken under similar spectrographic conditions. The scale readings of the solar lines on the spectrograms of the planet were corrected for the relative velocity of Venus and the earth. The average difference between these adjusted scale readings and those of the same lines on the sky spectrograms was then applied to all lines, thus bringing the readings of the solar lines into mean agreement on the two spectrograms. In the absence of absorption by oxygen in the atmosphere of Venus, the oxygen lines should now have the same scale readings in the two spectra; on the other hand, the presence of absorption should register as a displaced component of the oxygen lines, broadening or doubling them, according to the relative velocity. Spectrograms were taken with Venus east and west of the sun, the range in velocity corresponding to a Doppler

TABLE II

OBSERVATIONAL DATA

Plate	Subject	1921	P.S.T.	z sec z	Velocity E−V	Doppler Shift	Δλ	Lines Meas.	Refer. Lines	Phase
					km	A	A			
V 274....	Venus	Feb. 24	5^h36^m–9^h00^m	67° 2.56	−12.87	−0.254 −0.270	+0.015 −0.010	water vapor a, oxygen	arc sun	98°.0
275....	Sky	Feb. 25	4 30	76° 4.13	....	....	+0.003 −0.001	water vapor a, oxygen	sun	....
276....	Venus	Feb. 25	5 40 −9 00	68° 2.67	−12.84	−0.253 −0.269	+0.014 −0.004	water vapor a, oxygen	arc sun	98°.7
277....	Sky	Feb. 27	4 58	81° 6.39	....	....	−0.006 +0.001	water vapor a, oxygen	sun	....
278....	Venus	Feb. 27	5 37 −9 00	68° 2.67	−12.79	−0.252 −0.268	−0.012 −0.015	water vapor a, oxygen	arc sun	100°.3
279....	Sky	Feb. 28	3 49	68° 2.67	....	....	−0.009 −0.013	water vapor a, oxygen	arc	....
280....	Sky	Feb. 28	4 10	71° 3.07	....	....	−0.014 −0.021	water vapor a, oxygen	sun	....
282....	Venus	Feb. 28	7 20 −9 05	76° 4.13	−12.76	−0.252 −0.268	−0.005 −0.006	water vapor a, oxygen	sun	101°.1
285....	Venus	Mar. 1	7 19 −9 00	76° 4.13	−12.73	−0.267	−0.005	a, oxygen	sun	101°.8
289....	Venus	Mar. 3	6 16 −9 00	69° 2.79	−12.69	−0.249	+0.026	water vapor	sun	103°.4
301....	Venus	Mar. 24	5 55 −8 35	70° 2.92	−10.99	−0.252	+0.00	B, oxygen	sun	125°.1
303....	Venus	Mar. 27	5 45 −8 38	71° 3.07	−10.38	−0.238	0.00	B, oxygen	sun	129°.0
359....	Venus	Aug. 13	15 0 −23 0*	31° 1.17	+12.44	+0.286	−0.00	B, oxygen	sun	67°.1

* Exposure interrupted; total 7^h10^m.

shift of 0.538 A. The results of the comparison are given in the eighth column of Table II, where the $\Delta\lambda$ for these plates is the difference Venus *minus* sky.

OXYGEN

Five spectrograms of Venus were taken in the region of the a band when the relative velocity of Venus and the earth was -12.8 km, the corresponding Doppler shift being 0.268 A to the violet, an amount sufficient to separate completely the terrestrial components from those of Venus. Although some solar lines of intensity 000 are faintly visible on these spectrograms, no lines are observable where they should appear if produced by oxygen in Venus' atmosphere. The measured wave-lengths of the oxygen lines present on the spectrograms of Venus and the sky and the magnitude of their deviations from the wave-lengths of Table I show, moreover, that the oxygen lines produced by the earth's atmosphere in the spectrum of the planet are not measurably shifted by blending with lines originating in the atmosphere of Venus and wide enough to overlap those of terrestrial origin.

The B band is producible by a much smaller quantity of oxygen than the a band and therefore furnishes a more sensitive test. King has recently shown that 39.4 m of air at 72 cm pressure, equivalent to 8 m of oxygen, give the lines of the B band faintly.[1] His solar spectrograms indicate that the lines of the B band produced by 39.4 m of air are comparable in intensity with solar lines of intensity 00 on the Rowland scale. On the spectrograms of Venus in this region lines of intensity 1 have about the same visibility as the limiting lines on King's spectrograms. To obtain an estimate of the length of an oxygen column which would produce the lines of the B band with an intensity of 1, we have made use of a valuable paper by Jewell[2] on the variation of the intensity of water-vapor lines with the quantity of water vapor traversed and of the change in intensity of the oxygen line λ 6287.953 with zenith distance. Jewell determined the intensities of Fraunhofer lines—000 to 6— in terms of the Fe line λ 5930.406. Graphs from these data show that the inten-

[1] *Mt. Wilson Contr.*, No. 232; *Astrophysical Journal*, **55**, 411, 1922

[2] *Astrophysical Journal*, **4**, 324, 1896.

sity intervals from ooo to 6 are roughly equal, and that the intensity of the water-vapor line λ 5919.860 is strictly proportional to the amount of water vapor traversed (Figs. 1, 2). A similar proportionality is shown to hold for the oxygen line λ 6287.953 and is assumed to hold for the oxygen lines of the B band. The interval from oo to 1

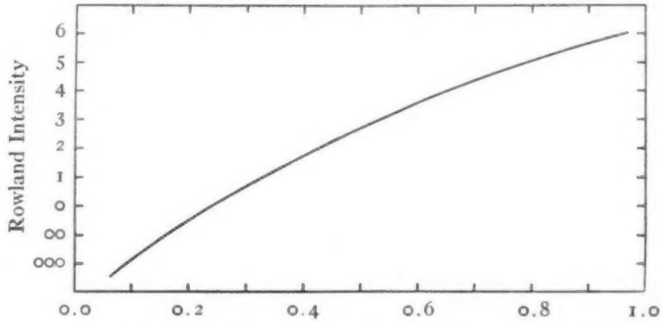


FIG. 1.—Rowland intensities in terms of λ 5930 taken as unity

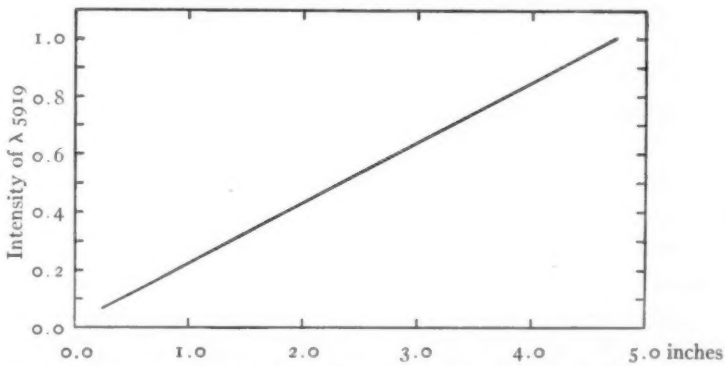


FIG. 2.—Intensity at the zenith of λ 5919 (wv) in terms of λ 5930 (Fe) for various amounts of precipitable water in the atmosphere expressed in inches.

is approximately 0.16 of the intensity of the Fe line λ 5930. This is equivalent to an increase in the length of path of 16 per cent. Hence the lines of the B band would probably appear on the spectrogram of Venus if the absorbing column contained the equivalent of 9.2 m of oxygen under ordinary conditions of pressure.

Two spectrograms of Venus showing the B band, March 24 and March 27, 1921, were taken when the relative velocity was −10.68 km, the equivalent Doppler effect being −0.245 A. A

similar spectrogram was taken August 14, when the relative velocity was +11.36 km, the Doppler displacement being +0.286 A. With the scale of 3 A per millimeter, the doublets of the B group are widely separated, but in the interspaces no lines are visible, where lines due to oxygen absorption in the atmosphere of Venus should appear, though solar lines of intensity 0 are present. Whether lines originating in the planet would be completely separated from the terrestrial lines by shifts of 0.264 A and 0.286 A depends on the widths of the lines. It is evident that at most only a small amount of oxygen is traversed in the atmosphere of Venus, and lines produced by it would be faint and narrow. On a plate of the B band taken by King with an air path of 39.4 m, the mean width of the single lines is 0.19 A. The mean width of these lines on plate V 359 is 0.27 A, and the Doppler shift +0.29 A, so that the edges of the lines would be separated by 0.06 A. Spectrograms V 301 and V 303 are not so suitable for this purpose, as the continuous background is fainter and the lines somewhat wider and the Doppler shift smaller than for V 359. When, however, the positions of the oxygen lines in these spectrograms are compared with their positions in the sky spectrograms, the measures show a displacement of 0.003 A to the red, whereas the Doppler shift would be 0.245 A to the violet. On V 359, the measured displacement is 0.002 A to the violet, with a possible Doppler shift of 0.285 A to the red. If, therefore, for any cause the lines of the B band produced by small amounts of oxygen on Venus are somewhat wider than those produced in the laboratory, the results show that there is no measurable influence of overlapping of the edges of the terrestrial lines by undetected Venus components.

The length of the path traversed through Venus' atmosphere in a layer of a given radial depth depends upon the phase angle of the planet and the position on the disk of the point from which the light is observed. For a point on the line perpendicular to the terminator and limb, whose distance from the limb in units of the length of that line is d, the zenith distances of the earth, z_e, and of the sun, z_s, are given by

$$\sin z_e = 1 - d - d \cos i ,$$
$$z_s = i - z_e$$

where i is the phase angle. Table III gives the total length of the path traversed by the solar beam for different phase angles and different distances from the limb in units of the radial depth. The secant equation used here is of course not applicable just at the limb and terminator.

TABLE III

$\sec z_e + \sec z_s$

d		i		
		70°	100°	130°
(Limb)	0.0..........	∞	∞	∞
	0.1..........	3.0	3.7	5.5
	0.2..........	2.6	3.2	4.8
	0.3..........	2.4	3.1	4.8
	0.4..........	2.5	3.2	5.0
	0.5..........	2.6	3.5	5.6
	0.6..........	3.0	4.0	6.6
	0.7..........	3.5	5.0	8.2
	0.8..........	4.7	6.9	11.6
	0.9..........	8.1	12.9	22.9
(Terminator)	1.0..........	∞	∞	∞

At the middle point of the visible disk, corresponding to the center of the spectral band, the path for plates V 301 and V 303 was 5.2 times the radial depth of the layer penetrated, and for V 359 it was 2.5 times the depth. On the basis that lines of intensity 1 would be produced by 9.2 m of oxygen, it follows for V 301 and V 303 that the quantity of oxygen traversed did not exceed the equivalent of a column 2 m long under standard conditions. For light from near the terminator the path traversed in Venus' atmosphere is much longer. With the slit normal to the terminator, as in these observations, one edge of the spectrum corresponds to this longer path. For spectrograms V 301 and V 303 the path for the light at the point halfway between the center and edge of the spectrum was 7.5 times the radial depth of the layer. The edge of the spectrum shows no trace of oxygen lines due to absorption in the planet's atmosphere. Absorption capable of producing lines of intensity 1 would be expected if an oxygen layer equivalent to 1 m under normal pressure had been traversed in the passage through the atmosphere of Venus. As the oxygen in the earth's

atmosphere is equivalent to a column 1500 m long, it follows that the oxygen in the path through Venus' atmosphere was less than one-thousandth of that in the terrestrial atmosphere.

WATER VAPOR

Five spectrograms of Venus taken when the relative velocity of Venus and the earth was -12.8 km, the corresponding Doppler shift being -0.252 A, form the basis of the investigation on water vapor. The relative velocity was sufficient to separate completely lines produced by absorbing gases common to both atmospheres. No traces of lines due to the planet's atmosphere are discernible on the spectrograms.

An upper limit for the amount of water vapor in the atmospheric layer on Venus penetrated by the solar beam may be found by the following considerations. Spectrogram V 282 was taken at mean zenith distance 76°. The average quantity of precipitable water above Mount Wilson is 0.69 cm[1] (the mean for the days of observation was 0.72 cm). The water-vapor line λ 5919.860 was of intensity about 5 on the Rowland scale, and telluric water-vapor lines of intensity 000 and 00 can be identified on the spectrum of Venus. Had a like quantity of water vapor been present in the planet's atmosphere above the reflecting surface, the intensity of the component of λ 5919.860 due to Venus should have been about 4 as against 5 for the terrestrial line, since the water-vapor masses traversed would have been 2.5 cm for Venus and 2.9 cm for the earth, the two respective paths being 3.6 and 4.1 times that for zenith distances 0. For the point half-way between the center of the disk and the terminator the water mass traversed would have been 4.2 cm, corresponding to solar intensity 7 for the water-vapor line λ 5919.860. These results are deduced from the graphs of Jewell's data. Terrestrial water-vapor lines of 00 intensity are easily seen on the plate, but no line is visible 0.25 A to the violet of the terrestrial line λ 5919.860, in which position 0.7 cm of water above the apparent surface of Venus would have produced a line of intensity 7. As the water vapor traversed in the atmosphere of Venus was not sufficient to produce the line λ 5919.860 with intensity 00, there must have been

[1] *Annals of the Astrophysical Observatory*, **3**, 189, 1913.

less than 1 mm of water in the layer of the planet's atmosphere traversed by the solar beam, for a line of intensity 00 is produced by 16 per cent of the water vapor necessary for a line of intensity 7.

Measurements of the wave-lengths of eight water-vapor lines were made on the five spectrograms of Venus and on four of the sky. The wave-lengths in Table I were obtained from solar spectrograms with a dispersion four times that used in this investigation. For these closely crowded lines the mean excess of 0.008 A shown by the Venus measures in Table II, when they are compared with the values given in Table I, represent probably the systematic errors associated with the measurement of spectral lines not well separated and varying in intensity from time to time as is characteristic of atmospheric lines. The difference is positive, whereas it should be negative if there were a measurable effect due to partial superposition of lines produced by water vapor in Venus' atmosphere.

DISCUSSION

These observations indicate that the previous spectroscopic evidence for oxygen and water vapor in the atmosphere of Venus, depending upon visual observations of a change in line intensity, is not reliable, that in fact there is no acceptable spectroscopic evidence of the presence of either. On the other hand, they do not show the complete absence of water vapor and oxygen from the planet's atmosphere, but that, to the depth penetrated by the solar beam, they are not present beyond a definite low limit.

Previous to Russell's[1] investigation in 1898 the accepted view as to the extent of Venus' atmosphere made it much denser than our own.[2] Russell showed that the prolongation of the cusps, formerly attributed to refraction, was mainly due to diffuse reflection of light by the planet's atmosphere. He concluded that the horizontal refraction at the apparent surface cannot exceed 12′ as against 34′ for the earth, and that there is no satisfactory evidence that the atmosphere of Venus at the apparent surface is more than one-third as dense or extensive as the earth's at sea-level. The entire height above the apparent surface he thinks may be thirty miles as com-

[1] *Astrophysical Journal*, **9**, 284, 1899.

[2] Young, *Manual of Astronomy*, p. 356, 1912.

pared with the earth's forty, thus implying a pressure at this surface of perhaps one-tenth that of the earth's atmosphere at sea-level. He attributes the prolongation of the cusps visible in daylight to a hazy layer of fog or dust lying below the 4100-foot level, a haze-bank whose upper boundary presents a more or less abrupt decrease in haziness, and whose lower portions may at times be obscured by low mountains, evidenced by irregularities in the thin circle of illumination on the side opposite the sun when Venus is at inferior conjunction.

A. W. Clayden[1] makes a strong presentation of the view that Venus has a moisture-laden atmosphere as dense and extensive as that of the earth. He decides on a long period of rotation, but one less than 225 days. He assumes that a high and heavy layer of pillared cumuli covers the cyclonic areas of low pressure and rising vapor, that over the regions of greater pressure the anticyclonic circulation results in lower detached masses of cumuli between which we see perhaps the shaded surface of the planet, and that a filmy veil of cirrus produces the prolongation of the cusps.

If, as suggested by Russell, the pressure at the visible surface is only one-tenth of an atmosphere, then the atmosphere of Venus is much poorer in oxygen than the earth's atmosphere, as the quantity of oxygen in Venus' atmosphere above the visible surface is less than the equivalent of 1 m, while in our atmosphere above the level of similar pressure there are 104 m of oxygen. If the reflection takes place at the upper surface of a 4100-foot haze-bank, which Russell suggests may be partially cut off at times by low mountains and therefore may not be far above the actual surface of the planet, then the oxygen above this low level is less than one-thousandth of that in the earth's atmosphere above Mount Wilson, elevation 1739 m.

If, on the other hand, it be assumed, as by Clayden, that the atmosphere of Venus is similar in composition and extent to ours, it follows that the reflected sunlight has not penetrated to a point less than 43 km from the real surface of the planet and has traversed but an insignificant path through its atmosphere, since in the earth's atmosphere there is above the 43 km level the equivalent of a meter

[1] *Monthly Notices*, **69**, 195, 1909.

of oxygen under normal pressure, an amount capable of registering on our spectrograms. The question of the depth to which the solar beam penetrates the planet's atmosphere and the probability that it reaches a point much nearer the real surface than 43 km are considered in detail in the discussion of water vapor.

TABLE IV

AMOUNT OF OXYGEN ABOVE A GIVEN ELEVATION

Elevation	Pressure	Oxygen
km	mm	m
0	760	1490
5	405	722
10	201	324
15	90	135
20	41	54
25	19	23
30	8.6	10.0
35	3.8	4.2
40	1.6	1.76
45	0.85	0.73
50	0.40	0.30

Any conclusion as to water vapor must rest upon the nature and temperature as well as the elevation of the apparent surface at which diffuse reflection takes place. The high albedo has been considered evidence of a cloudy surface, but the equally high reflecting power of Vesta indicates that this evidence is not definitive, since for Vesta there is no question of a cloudy surface. Russell[1] gives a Bond albedo of 0.59 for Venus. Using 65 per cent as the reflecting power of a cloudy surface, Abbot and Fowle[2] found 0.60 as the corresponding albedo for a cloud-covered earth. Aldrich[3], under more favorable conditions of observation, obtained 78 per cent as the reflecting power of clouds, from which the albedo calculated in the same manner is 0.75 for a cloud-covered earth. Russell says that the Bond albedo A should not be used for comparison with the observed reflecting power of terrestrial substances, but a quantity p, which he defines in an article on the albedo of

[1] *Loc. cit.*, p. 190.

[2] *Annals of the Astrophysical Observatory*, **2**, 161–163, 1908.

[3] *Smithsonian Misc. Collections*, **69**, 10, 1919.

planets and satellites.[1] For Venus p is 0.49 while for clouds it is 0.78. Both the reflecting power and the Bond albedo are considerably less than would be expected if Venus is completely cloud covered.

The conditions of Venus' atmosphere depend in large measure upon the period of rotation, the temperature, and the temperature gradients, all undetermined quantities. The period of rotation is quite probably not less than fifteen days, the limit shown by Slipher's observations with which our own agree. With a period of rotation equal to that of the orbital revolution, it is reasonable to assume, as Arrhenius suggests,[2] that water vapor and all other constituents of an atmosphere would collect and be congealed on the side of the planet turned away from the sun where a temperature near absolute zero would prevail. But the illumination of the whole circumference of the disk when Venus is at inferior conjunction, attributed to scattering and refraction, shows the presence of an atmosphere of considerable extent. These considerations give grounds for assuming that the period of rotation is shorter than that of the orbital revolution and hence is between the extremes of 15 and 225 days. For an intermediate period of rotation, an atmosphere would be present over the region of insolation, subject probably to violent disturbances near the terminator. This would be in harmony with Quenisset's[3] observations, both photographic and visual, of extensive and rapid changes in the obscuring envelope, indicating a shallow atmosphere.

The region of maximum cloudiness in the terrestrial atmosphere is mainly below 4000 m,[4] coinciding roughly with the average altitude of the zero isotherm, which is 3000 m.[5] The corresponding region of cloudiness in the atmosphere of Venus would be where the temperature and humidity conditions are similar. According to Abbot and Fowle,[6] the mean temperature of the earth's surface is

[1] *Astrophysical Journal*, **43**, 191, 1916.

[2] *Worlds in the Making*, p. 61, 1908.

[3] *Comptes Rendus*, **172**, 1645, 1921.

[4] Humphreys, *Physics of the Air*, p. 309, 1920.

[5] Hann, *Handbook of Climatology*, p. 250, 1903.

[6] *Annals of Astrophysical Observatory*, **2**, 173–175, 1908.

287°.2 A, but the temperature of the true radiating surface of the earth as a planet is 263° A, this surface being chiefly the water-vapor layer at an elevation of 4000 m or more above sea-level. The elevation of a water-vapor radiating surface on Venus, however, would probably be greater owing to higher temperature. Using 2.5 calories as the solar constant, Poynting[1] gives the average temperature of an ideal planet at Venus' distance from the sun as 342° A, and at the earth's distance as 290° A. With 287°.2 A for the temperature of the earth the temperature of Venus becomes 337° A.

If the surface temperature of Venus be taken as 337° A and a temperature gradient of 6° per kilometer be assumed in Venus' atmosphere, such as obtains on the earth, the elevation of the zero isotherm would be 11 km, that of a water-vapor radiation surface 12 km, and that of the isothermal layer 19 km, which are to be compared with 3, 4, and 11 km for the earth.

The conditions at Mount Wilson offer favorable opportunities to study the increase in the relative humidity of the air above a cloudy surface. The velo cloud,[2] as the Spanish called the "high fog" of southern California, often covers the valleys to a height of 1200–3000 feet, especially during the months of May and June. For nine clear days in May and eight in June, 1921, the mean vapor pressure on Mount Wilson was 3.25 and 4.05 mm, respectively, while for nine days in May and twenty-one in June, with clear sky above and the velo cloud 4000 feet below and two to five miles distant, the respective vapor pressures were 4.65 and 5.28 mm, about 35 per cent greater. In the case of Venus' atmosphere with a water-vapor content sufficient to form a continuous layer of cumulus clouds exposed to nearly twice the intensity of the solar radiation at the earth, the water vapor above the dense cloud layer would probably exceed the increase at Mount Wilson when velo clouds are 4000 feet below and several miles away. This increment is two or three times the quantity our spectrograms should record. Granting the probable circulation in the planet's atmosphere suggested by Clayden, it seems also probable, especially over the anticyclonic

[1] *Collected Scientific Papers*, p. 315, 1920.

[2] Carpenter, *The Climate and Weather of San Diego*, 1913.

areas, that the solar beam would penetrate a region of the atmosphere where water vapor would be encountered in detectable quantity. The spaces not only above, but especially between the columnar cloud masses, must be more or less moisture laden, since, under the powerful solar radiation, evaporation and convection into the neighboring drier regions would take place as over the velo clouds, and under the downward anticyclonic flow, the clouds would be carried into regions of higher temperatures and more or less dissipated. The results would be an atmosphere of high humidity over extensive areas more or less open to observation, as the high cirrus assumed by Clayden would offer little obstruction.

If the dusky markings upon which investigators have based the observations interpreted as showing short rotation periods are actual surface features of the planet, or if they are a transient thinning of a cloudy envelope, it is probable that we there see down to levels at which the humidity would be high in an atmosphere so heavily moisture laden that the planet is enveloped in a blanket of clouds. Spectrogram V 274 was taken eight hours later than Quenisset's photographs showing extensive dusky areas, but no traces of water-vapor lines are discernible on any part of the spectral band.

If the brilliant white spots from which the long rotation period has been deduced really belong to the planet's surface, and, as suggested, are snow-covered mountain peaks extending into the upper atmosphere, it seems probable that water vapor would be spectroscopically detectable by its absorption lines were they to stand free from the corresponding terrestrial lines, as they would in our spectrograms. It is to be considered also that the spectrograms were taken in light of long wave-length, for which scattering is relatively small and the consequent penetration into a hazy atmosphere great. In the case of the oxygen lines, near λ 6900, the depth from which the light is reflected is well below the apparent visual surface.

If, however, the reflecting surface consists of a permanent layer of cirro-stratus, the quantity of water vapor traversed by the reflected beam would be small, as cirro-strati are formed in the upper troposphere where the temperatures are very low. These low temperatures practically insure that cirro-strati consist of ice crystals, but as at such low temperatures there is little water to con-

dense, their structure is necessarily more or less tenuous and fibrous and consequently the reflected light will have penetrated the layer to a great depth. Above such a cloud-surface there would be the thinner cirrus to which the prolongation of the cusps is due, and the still higher atmosphere which produces by refraction the illumination around the planet's circumference observed at inferior conjunction.

Reflection from a layer of cirri gives the shortest possible path for the light in the planet's atmosphere. The water vapor above the cirrus level may be insufficient for detection by observations on the lines in the rain-band. The spectrographic test for oxygen by observations on the B band is, however, much more sensitive. In the earth's atmosphere above the cirrus level there is the equivalent of 274 m of oxygen at sea-level; above 19 km, the elevation assumed for the cirrus level on Venus, there is still the equivalent of 65 m, but our observations show that oxygen on Venus above the reflecting surface is less than 2 per cent of this amount.

It is of interest to consider some alternatives to water-vapor clouds. It is possible that a very small quantity of water vapor would produce an impenetrable haze-bank if the atmosphere of Venus contained minute hygroscopic centers of condensation capable of producing cloud particles in an atmosphere where the humidity is much below that which otherwise would be essential to cloud formation.[1] With such a hazy atmosphere on Venus, analogous to but denser than the dry-weather haze that sometimes obscures the valley or lies along the lower reaches of the distant mountains viewed from Mount Wilson, the surface of the planet might appear much as we see it. To an observer on the planet's surface transmitted light would be rich in long wave-lengths, as to us when the earth's atmosphere is laden with volcanic dust. To an outside observer the color would depend upon the proportion of the light scattered in its atmosphere to that reflected from its surface.

Under the probable extremes of temperature on the opposite hemispheres of Venus it is conceivable that the violent atmospheric circulation would cause clouds of dust to be permanent features of

[1] Humphreys, *Physics of the Air*, p. 92, 1920.

the planet's atmosphere, dust composed of highly reflecting material such as that to which the high reflecting power of Vesta is due. Wilsing and Scheiner[1] give for liparitic pumice a reflecting power of 0.56, for rock salt 0.44, and for granular limestone 0.42. These are comparable with 0.49 for Venus and 0.48 for Vesta. It is to be observed, however, that the presence of two of these substances would imply the action of water at the time of their production or segregation. The dust storms that sometimes prevail for days over the deserts and occasionally sweep through the mountain passes suggest vividly the possibilities on a dry and wind-swept planet.

Whether or not light reflected from the continually evaporating surfaces of clouds produces sufficient water vapor to give its absorption lines with observable intensity is a question whose answer would aid greatly in determining the character of the apparent surface of Venus and in solving the puzzling problems of its atmosphere. We hope later to obtain more observational data as to the humidity around and above clouds, and to extend the spectrographic observations to the water-vapor band near λ 7200, as the lines of this band are producible by smaller amounts of vapor and would furnish a more sensitive test of its presence. Other and quite as important information would be obtained from the relative color indices of Venus and terrestrial clouds and from direct photographs through violet and infra-red filters[2] as used by R. W. Wood for Jupiter and Saturn. These would show the apparent surfaces at widely different levels and offer the possibility of reaching the actual surface through the thinner portions of the atmospheric veil.

It has been too easily assumed, perhaps, that the atmospheric conditions on our nearest planetary neighbors are similar to those on the earth, and that on Venus development has followed along similar lines and by like stages as on the earth. It was long ago suggested by Koene,[3] of Brussels, that all free oxygen may have been formed from carbonic acid in the air. Arrhenius says that we may take it as established that the masses of free oxygen in the air and of free carbon in the sedimentary strata approximately corre-

[1] *Potsdam Publications*, **20**, Part IV, 1909.

[2] *Mt. Wilson Contr.*, No. 113; *Astrophysical Journal*, **43**, 310, 1916.

[3] *Mémoires de Chimie*, 1856.

spond to each other, and that probably all the oxygen of the air owes its existence to plant life.[1] That a similar production of oxygen has apparently not taken place on Venus suggests that some condition is wanting. Possibly a deficiency of water has prevented or hindered the freeing of oxygen through vegetation, or it may be that the exacting conditions for the origin of life have not been satisfied so that the existing atmosphere may consist of other permanent or semipermanent gases such as nitrogen or carbon dioxide.

Mount Wilson Observatory
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[1] *Worlds in the Making*, pp. 58, 59.

THE SOURCE OF LUMINOSITY IN GALACTIC NEBULAE[1]

By EDWIN HUBBLE

ABSTRACT

Relation of the luminosity of galactic nebulae to the magnitudes of associated stars. (1) *Diffuse nebulae.* If, as has been suggested, these nebulae owe their luminosity to radiations from associated stars, then the angular extent of nebulosity should increase with the apparent brightness of the stars. More precisely, if we assume that the clouds of nebulosity are illuminated by stellar light whose intensity varies inversely as the square of the distance from the stars; that each part of a nebula reflects or re-emits, without change in actinic value, all the starlight intercepted by it; and that the light from the stars themselves reaches us undimmed by absorption, then the square of the maximum angular extent of nebulosity a, for an exposure E, should be proportional to E times the apparent luminosity of the star I, or $a^2/E=a_1^2/60=I\times$ const., or $m+5\log a_1=B$, where a_1 is the angular distance reduced to a uniform exposure of 60^m. For a Seed 30 plate and a focal ratio of 5 to 1, B is equal to 11.6 or 10.6, according as the line joining the star to the faint nebulosity is perpendicular to the line of sight or has the mean inclination corresponding to random direction. To test this equation, data for eighty-two nebulae were plotted, $\log a_1$ against m. The points lie near the line: $m+4.9\log a_1=11.0$, thus showing that the inverse-square law holds closely and that the other assumptions are approximately correct. The mean deviation of the points from the line, over an interval of 14 magnitudes, is only $\pm 0^m.8$; but thirteen points lie above the limiting theoretical curve. The probable explanation is that in these cases the apparent brightness of the stars is diminished through absorption by nebulosity between the star and the observer, that is, the third assumption is not wholly justified. This is borne out by the fact that these stars show a color excess which is usually greater than the residual magnitude. While most of the nebulae have continuous spectra, in eighteen cases the spectra are emission, and thus differ widely from the spectra of the associated stars. Since the corresponding points show no systematic deviation from the theoretical curve, the luminosity must be due to re-emission of light of the same total actinic value, rather than to reflection, and this suggests the possibility that the light producing the continuous spectrum is also re-emitted. The great loop in Cygnus is a conspicuous exception, since no sufficiently bright star is in evidence. The star, perhaps, is hidden or dimmed by nebulosity. Reduced to absolute magnitudes, the foregoing equation becomes $M+5\log l=2.5\log E-5.52$, where l is the linear distance in parsecs from star to nebulosity and E is in minutes. For Rigel, $M=-5.6$, an equivalent exposure of thirteen hours shows nebulosity, presumably made luminous by it, extending to $12^\circ.5$, or ninety-three light-years. (2) *Planetary nebulae.* When certain systematic corrections suggested by observations made at Mount Wilson are applied to data given by Curtis for the limiting exposure times and the apparent magnitudes of the nuclei for fifty-six objects, the angular distance A_1 of the brightest details of luminosity from the central stars, reduced to uniform exposure time on the assumption of an inverse-square law of luminosity with respect to the central stars as sources, varies with the photographic magnitude of the central stars according to the equations: $m+(5.3\pm0.4)\log A_1=17.9$, or $m+5\log A_1=17.6\pm0.2$. The first of these is the correlation curve derived from the data; the second, the curve having the theoretical slope which best represents the plotted points. The correlation between m and $\log A_1$, however, is not nearly as close as in the case of the diffuse nebulae. The residuals, for instance, seem to be related to the type of planetary, averaging $+1.8$ for the ring type, and -1.5 for globular

[1] *Contributions from the Mount Wilson Observatory*, No. 250.

objects. The corresponding theoretical equation, based on the assumption stated above for diffuse nebulae, but for an exposure time of six seconds, is: $m+5 \log A=13.6\pm 0.3$. Hence, the observed luminosity per unit area is about four magnitudes brighter than the theoretical. The discrepancy for these nebulae may be due to a difference between the spectra of the nebulosity and of the nuclei, the latter having relatively much more energy in the ultra-violet; or it may indicate that the radiations which excite the luminosity are chiefly corpuscular.

Revised photographic magnitudes of the nuclei of planetary nebulae.—The values given by Curtis for fifty-six objects seem to require a systematic correction of

$$+0.173m-3.37.$$

New cometary nebulae at $\alpha=6^h3^m$, $\delta=+18°42'$ (1920) shows striking similarity to the variable nebula N.G.C. 2261, although variability has not yet been established.

I. DIFFUSE GALACTIC NEBULAE

This paper gives the results of an attempt to test the theory that stellar radiations are the source of luminosity of galactic nebulae. V. M. Slipher[1] found that in the case of Merope, Maia, ρ Ophiuchi, N.G.C. 2068 and N.G.C. 7023 the absorption spectrum of the nebulosity agreed, approximately at least, with the spectra of the brightest involved stars. On this ground he suggested that these nebulosities at least are shining by reflected light from the involved stars. Hertzsprung[2] made a quantitative test of this suggestion in the case of the Pleiades nebulosity by determining the surface brightness at several points in the nebulosity and comparing the results with those derived by spreading the light of the stars over spherical shells of radii equal to the distances from the stars to the points of nebulosity whose surface brightness he had measured. He found the nebulosity to be from 4 to 5 magnitudes fainter than would be expected on a theory of total reflection, and concluded that, if a suitable albedo were assumed, the reflection of star light could account for the nebular luminosity. H. N. Russell[3] has accepted the idea of reflection for diffuse nebulae with continuous spectra, and has advanced the theory that galactic nebulae with emission spectra, diffuse as well as planetaries, are excited to luminosity by radiations from involved or neighboring stars.

The writer has found that particular stars can be selected which are obviously involved in or conspicuously associated with prac-

[1] *Lowell Observatory Bulletin* No. 55, 1912; 75, 1916; *Astronomical Society of the Pacific*, **30**, 63, 1918; **31**, 212, 1919.

[2] *Astronomische Nachrichten*, **195**, 449, 1913.

[3] *Proceedings of the National Academy of Sciences*, **8**, 115, 1922; *Observatory*, **44**, 72, 1921.

tically every known diffuse nebulosity. The spectra of these stars bear a definite relation to the spectrum of the associated nebulosity, such that if the stellar spectra are earlier than B1, the nebular spectrum is emission, and, if the stellar spectra are later than B1, the nebular spectrum is continuous. Nebulosity associated with stars of the critical type B1 usually possesses a spectrum of mixed characteristics. The transition in nebular spectra from emission to continuous, although rapid, is quite smooth. It is not abrupt. The only pronounced exception to the rule of spectral relation is the possibility that stars with highly enhanced spectra of the α Cygni type may be associated with nebulosity whose spectrum is predominantly emission. α Cygni itself and B.D. $-22^\circ 4510$, of the same type, appear to be associated with N.G.C. 7000 and N.G.C. 6514, respectively. Bright-line stars associated with emission nebulae are the exception and not the rule. Wolf-Rayet stars associated with diffuse nebulosity are rare. In the northern skies only two cases are known—N.G.C. 2359 and 6888. Both show some affinity with planetaries or with nova phenomena.

This definite spectral relation suggests that nebular luminosity depends to a certain degree at least on stellar radiations, and an attempt has been made to test this quantitatively. The method employed was to determine the surface brightness of diffuse nebulosities giving continuous spectra, by means of the limiting exposure times necessary to just register the nebulosity under standard conditions. Preliminary results from a small number of objects demonstrated that the brightness and extent of the nebulosity are definitely related to the magnitude of the associated stars, and that existing photographs could be used to establish the relationship. Accordingly Table I was constructed from an examination of the plates already available. In the first column is given the catalogue designation of the nebulae; in the second column, the spectral type of the nebulae, E and C for observed emission and continuous spectra, e and c for the same characteristics, not observed, but inferred from the spectral types of associated stars, according to the rules given in the preceding paragraph; in the third column the apparent photographic magnitudes, m, of the associated stars, combined, if a group of stars appears; in the

fourth, the spectral type of the associated stars; in the fifth, the color excess[1] of these stars; in the sixth, the exposure time, E, in minutes, of the plates examined; in the seventh, the greatest angular distances, from the stars, in minutes of arc, at which nebulosity could be detected. For a group of stars involved in nebulosity, a was measured from the mean position of the stars. In most cases of this sort the extent of the groups of stars is small compared to the extent of nebulosity. In the eighth and ninth columns are given the quantities log E and log a; in the tenth, the corrected quantity log a_1, representing log a reduced to a uniform exposure time of 60 minutes; in the eleventh, the designations of the stars whose magnitudes are given in the third column; in the twelfth column the instruments employed in obtaining the photographs examined.

The data are fairly homogeneous, as the plates used were all Seed 30's, most of them taken with the 60-inch and 100-inch reflectors, both of focal ratio 1 to 5. Several were taken with a Tessar Ic lens, called "Kodax" in the eleventh column, and with a 10-inch Cooke astrographic lens, both of which have focal ratios 1 to 4.5. In these cases the values of log E were corrected by the quantity $+0.12$ in order to reduce them to the Newtonian reflector system. A few measures are from photographs by Barnard, made with the 10-inch Bruce lens of focal ratio 1 to 5, and designated "Barnard" in the twelfth column. A small correction should be applied to log E in these cases in order to allow for the light cut off by the Newtonian flat, but the amounts and the numbers are so small that they have been disregarded.

The photographic magnitudes for stars brighter than 6.0 are from visual magnitudes found in the *Harvard Annals*, to which have been applied both color indices and color excess. Magnitudes of the fainter stars are from polar comparisons made by the writer, and are on the Mount Wilson scale.

The quantities used in the following discussion, m, a, and E, are quite independently determined, and there is no reason to suspect

[1] The color excesses are taken from *Mt. Wilson Contr.* No. 187; *Astrophysical Journal*, **52**, 8, 1920; and from further unpublished measures by Seares and Hubble. The latter values are provisional. Final values will appear in a joint paper to be published shortly.

TABLE I

Object	Spec. Nebula	m	Spec. Star	CE	E	a	Log E	Log a	Log a_1	Star	Instrument
					min						
B.D. +64°13	c	10.4	B2	0.65	100	1'.5	2.000	0.176	0.065	....	60-inch
N.G.C. 281	E	7.3	Oe5	0.10	210	14	2.322	1.146	0.874	B.D. +55°191	60-inch
I.C. 59, 63	E	2.2	B0p	0.00	60	33	1.898	1.518	1.458	γ Cassiopeiae	10-inch
B.D. +60°596	c	9.7	B2	0.40	120	3	2.079	0.477	0.326	....	60-inch
N.G.C. 1333	C	10.9	B9p	0.30	180	4	2.255	0.602	0.364	....	60-inch
B.D. +31°597	c	8.6	K2g	0.00	60	2	1.778	0.301	0.301	....	100-inch
+29°565	c	9.6	A5	0.33	120	4.3	2.079	0.634	0.483	....	60-inch
Merope	C	4.0	B5	0.00	120	25	2.079	1.398	1.247	....	60-inch
Pleiades	c	1.2	....	0.00	615	270	2.789	2.431	1.926	Entire cluster	Barnard
I.C. 348	C	9.0	B6–A1	0.80	180	2.3	2.255	0.362	0.124	B.D. +31°643	100-inch
N.G.C. 1491	E	11.3	B0	1.00	120	1.8	2.079	0.255	0.104	....	60-inch
1499	E	3.7	Oe5	0.00	120	70	2.199	1.845	1.634	ξ Persei	10-inch
1514	E	9.7	Odp	0.70	90	1.8	1.954	0.255	0.167	B.D. +30°623	100-inch
I.C. 359?	C	13.5	K8d	0.00	240	0.4	2.380	−0.398	−0.699	....	100-inch
N.G.C. 1624	E	12.0	Oe5	......	120	1.5	2.079	0.176	0.025	....	60-inch
1788	C	10.2	B8	0.22	90	3	1.954	0.477	0.389	B.D. −3°1013	60-inch
I.C. 405	E	5.8	B0p	0.30	90	11.5	1.954	1.061	0.973	Boss 1249	60-inch
2118*a*	C	0.2	B8p	0.00	60	180	1.898	2.255	2.195	Rigel	10-inch
2118*b*	c	0.2	B8p	0.00	600	750	2.898	2.875	2.315	Rigel	Kodax
N.G.C. 1976	E	4.8	Oe5, B0	......	90	35	2.074	1.544	1.396	Trapezium	10-inch
1977	C+E	4.6	B1	......	120	20	2.079	1.301	1.150	c' Orionis	100-inch
I.C. 431	c	7.6	B2	0.17	120	3	2.079	0.477	0.326	B.D. −1°1001	100-inch
432	c	6.9	B2	0.05	120	4	2.079	0.602	0.451	−1°1005	100-inch
N.G.C. 2023	C	7.9	B2	0.30	90	6.8	1.954	0.832	0.744	−2°1345	60-inch
2024	E	1.4	B0	0.00	90	33	1.954	1.518	1.430	ζ Orionis	100-inch
I.C. 435	C	8.2	B3	0.20	90	4	1.954	0.602	0.514	B.D. −2°1350	60-inch
N.G.C. 2068	C	10.8	B5	1.00	120	4.5	2.079	0.653	0.502	+0°1177	60-inch
2071	C	10.8	B9	0.80	120	2	2.079	0.301	0.150	+0°1181	60-inch
Orion	c	1.4	B0	0.00	600	240	2.898	2.380	1.820	ζ Orionis	Kodax
Cometary Neb.	C	13.2	B1	......	60	1.5	1.778	0.176	0.176	....	100-inch
N.G.C. 2170	c	11.0	B1	1.02	90	1.5	1.954	0.176	0.088	B.D. −6°1415	60-inch

B.D. $-6°1417$	c	10.2	B5	0.55	90	3.1	1.954	0.491	0.403	......	60-inch
$-6°1418$	c	9.9	B5	0.65	90	1.5	1.954	0.176	0.088	......	60-inch
N.G.C. 2182	c	9.0	B3	......	120	2.0	2.079	0.301	0.150	B.D. $-6°1431$	60-inch
2183	C	14.0	B3	......	120	0.5	2.079	−0.301	−0.452	......	60-inch
2185	C	12.9	B3	......	120	1.3	2.079	0.114	−0.037	......	60-inch
B.D. $-6°1444$	c	10.8	B3	......	120	0.5	2.079	−0.301	−0.452	......	60-inch
N.G.C. 2175	E	7.3	Oe5	0.20	240	14	2.380	1.146	0.845	B.D. $+20°1284$	60-inch
2237	E	5.3	Oe5	0.00	180	35	2.375	1.544	1.246	*m* for group	10-inch
I.C. 446	C	11.8	B1	0.80	120	1.8	2.079	0.255	0.104	......	60-inch
447	C	7.7	......	......	120	7.5	2.199	0.875	0.664	*m* for group	10-inch
N.G.C. 2245	C	11.2	B1p	0.88	45	1.3	1.653	0.114	0.177	......	60-inch
2247	C	9.2	B2p	0.68	60	2.5	1.778	0.398	0.398	B.D. $+10°1772$	100-inch
2282	c	10.1	B2	0.35	90	2	1.954	0.301	0.213	$+1°1503$	60-inch
I.C. 2177	E	7.3	B0p	0.70	120	9	2.079	0.954	0.803	$-10°1848$	60-inch
B.D. $-12°1771$	C	8.1	B1	0.20	40	1.8	1.602	0.255	0.343	......	60-inch
$-23°5277$	c	9.2	B9	......	90	2.5	1.954	0.398	0.310	Triple star	60-inch
$-23°5296$	c	9.0	B1	......	90	2.5	1.954	0.398	0.310	......	60-inch
π Scorpii	C	2.8	B2	......	400	113	2.602	2.053	1.641	......	Barnard
I.C. 4592	C	3.9	B2	......	180	80	2.255	1.903	1.665	ν Scorpii	Barnard
4601*a*	C	7.1	B9, A0p	0.20	300	13	2.477	1.114	0.765	B.D. $-19°4358$, 9	100-inch
4601*b*	C	7.1	B8, A0	0.32	300	13	2.477	1.114	0.765	$-19°4361$	100-inch
σ Scorpii	c	3.2	B1	0.48	270	140	2.431	2.146	1.819	......	Barnard
I.C. 4604	C	5.1	B2, B3	0.50	270	60	2.431	1.778	1.451	ρ Ophiuchi	Barnard
4605	C	4.7	B3	0.12	270	50	2.431	1.699	1.372	22 Scorpii	Barnard
N.G.C. 6514	E	7.0	Oe5	......	90	10	1.954	1.000	0.912	B.D. $-23°13804$	60-inch
6523	E	5.4	Oe5, B0	......	90	12	2.074	1.079	0.931	*m* for group	10-inch
I.C. 4678	c	10.4	B3	0.50	60	2.3	1.778	0.362	0.362	B.D. $-23°13906$	100-inch
1274*a*	c	8.7	B1	0.35	60	2.7	1.778	0.431	0.431	$-23°13997$	100-inch
1274*b*	c	9.2	B1	0.35	60	2.5	1.778	0.398	0.398	$-23°13998$	100-inch
N.G.C. 6589	c	9.8	B2	0.42	90	1.9	1.954	0.279	0.191	$-19°4940$	100-inch
6590	c	10.5	B3, B5	0.40	90	2.1	1.954	0.322	0.234	$-19°4946$	100-inch
I.C. 1284	c	7.7	B2	0.50	90	5	1.954	0.699	0.611	$-19°4953$	100-inch
N.G.C. 6611	E	7.3	Oe5	......	90	6	1.954	0.778	0.690	*m* for group	60-inch
I.C. 1287	c	6.1	B2	0.62	270	25	2.551	1.398	1.011	Boss 4687	10-inch

TABLE I—*Continued*

Object	Spec. Nebula	m	Spec. Star	CE	E	a	Log E	Log a	Log a_1	Star	Instrument
N.G.C. 6726, 7	C	7.1	B9	0.10	60	3.8	1.778	0.580	0.580	B.D. −37°13023,4	100-inch
6729	C	10.0	Gp	......	60	1	1.778	0.000	0.000	R Cor. Aust.	100-inch
6888	E	6.7	Ob	......	180	8	2.255	0.903	0.665	B.D. +37°3821	60-inch
I.C. 4954, 5a	c	13.7	B3	1.10	60	0.4	1.778	−0.398	−0.398	......	60-inch
4954, 5b	c	12.2	B3	0.90	60	0.6	1.778	−0.222	−0.222	......	60-inch
4954, 5c	c	14.4	A3	0.55	60	0.2	1.778	−0.699	−0.699	......	60-inch
N.G.C. 6914a	C	8.9	B3	0.00	180	3.0	2.255	0.477	0.239	B.D. +41°3737	60-inch
6914b	C	9.2	B4	0.00	180	2.5	2.255	0.398	0.160	+41°3731	60-inch
γ Cygni, a	......	3.0	F8p	0.00	180	115	2.375	2.061	1.763	......	10-inch
γ Cygni, b	......	3.0	F8p	0.00	180	75	2.375	1.875	1.577	......	10-inch
N.G.C. 7000	C+E	1.8	A3p	0.00	180	220	2.375	2.342	2.044	α Cygni	10-inch
7023	C	7.4	B2p	0.55	120	6	2.079	0.778	0.627	B.D. +67°1283	60-inch
B.D. +57°2309	c	6.5	B2p	......	120	9	2.079	0.954	0.803	......	60-inch
+67°1332	c	8.2	B4	0.05	90	4	1.954	0.602	0.514	......	60-inch
N.G.C. 7129	C	10.5	B3	0.60	60	1	1.778	0.000	0.000	......	60-inch
I.C. 5146	c	10.4	B1	0.97	120	5	2.079	0.699	0.548	B.D. +46°3474	60-inch
N.G.C. 7635	E	8.5	Odp	0.20	90	6	1.954	0.788	0.690	+60°2522	100-inch

The actual E is given for 10-inch and Kodax plates, but 0.12 is added to the value of log E in these cases to reduce to the Newtonian reflector system.

Pleiades. Trumpler's value of m for the entire cluster is used, and a color index of −0.10 is applied. E and a are taken from Barnard's photograph of the exterior nebulosity. *Publications Lick Observatory*, **11**, Plate 15, 1913.

I.C. 359? A nebulous star north following the nebula at $\alpha = 4^h14^m$, $\delta = +28°5'$ (1920), described by Barnard in *Astrophysical Journal*, **25**, 223, 1907, and which may be I.C. 359.

I.C. 2118. "a" refers to the brighter portion north preceding Rigel. "b" is the extreme limit of the faint extensions that trail off south preceding for several degrees into Eridanus.

Orion. Refers to the great exterior loop around the belt and sword of Orion. The following side of the loop was used and was referred to ζ Orionis as the nearest of the extremely bright stars. This procedure is questionable in detail but probably permissible in determining orders of magnitude.

Cometary Nebula. A bright uncatalogued cometary nebula at $\alpha = 6^h3^m$, $\delta = +18°42'$ (1920).

π Scorpii. E and a are from Barnard's photographs reproduced in *Astrophysical Journal*, **23**, 144, 1906.

I.C. 4592, 4604, 4605. E and a are from Barnard's photographs reproduced in *Astrophysical Journal*, **31**, 8, 1910.

N.G.C. 6729. This is the variable nebula about R Coronae Australis. The star is also variable and the measures refer to a particular date.

large systematic errors. The range in m is from 0.2 to 14.4; in a from 0′.2 to 750′; in E from 40 min. to 790 min. The mean errors in m are estimated as about 0.1, and in a about 5 per cent. The actual observed E, is, of course, exact to the minute, but differences in sky conditions, state of mirrors, plates, and development, suggest a mean error in the equivalent exposure time of the order of 5 per cent.

The nebulosity fades more or less smoothly with increasing distance from the stars, and the points for which a was measured represent in general the limiting surface brightness which registers on the particular plate examined. If the exposures were all of the same duration, under uniform conditions, a comparison of m with a should then indicate the relation existing between nebular and stellar luminosity. If this relation were due to a reflection of star light, and the albedo of the nebulosity were constant, the relation would be expressed by $m+5 \log a=K$ where K is a constant depending on the albedo, the limiting surface brightness shown on the photographs, and the distribution of angles which the directions, stars to points, measured for a, make with planes perpendicular to the lines of sight.

As a first step in the investigation, m was plotted against log a, disregarding the differences in exposure times. This plot is shown in Figure 1. The extent of nebular illumination is clearly a function of the stellar luminosity, and the relation is approximately linear. A curve with a slope, or coefficient of log a, equal to 5 can be fitted to the plot in the form

$$m+5 \log a=11.9 \tag{1}$$

so as to represent the points fairly well. Considering the great differences in exposure times, and the fact that measures on the brighter stars are as a rule from longer exposures than those for the fainter stars, Figure 1 is pretty fair evidence that nebular luminosity obeys the inverse-square law with the associated stars as sources of illumination.

This assumption affords a simple method of referring the data to a homogeneous system by reducing log a to log a_1, corresponding to a uniform exposure E_0, or inversely, reducing log E to log E_1,

corresponding to a uniform angular distance a_0. Let I and I_0 be the minimum intensities registered in exposure times E and E_0,

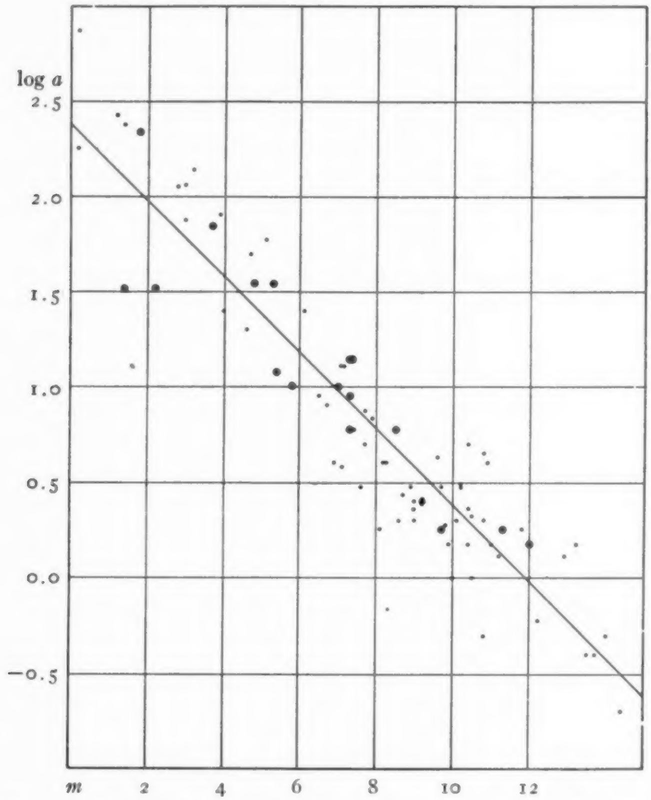


FIG. 1.—Photographic magnitudes of stars associated with diffuse nebulae (abscissae), plotted against the logarithm of the greatest angular extent of nebulosity from these stars (ordinates). The data are from Table I. Differences in exposure times of the plates examined are disregarded. The line represents the equation $m+5 \log a=11.9$. The coefficient of log a indicates that the nebular luminosity obeys the inverse-square law with respect to the stars as sources.

p Schwarzschild's exponent in the reciprocity law. Then, assuming the inverse-square law to hold rigorously for nebular illumination,

$$\left(\frac{a}{a_1}\right)^2=\frac{I_0}{I}=\left(\frac{E}{E_0}\right)^p \tag{2}$$

$$\log a_1=\log a+\frac{p}{2}(\log E_0-\log E) \tag{3}$$

likewise

$$\log E_1 = \log E + \frac{2}{p}(\log a_0 - \log a) \tag{4}$$

The magnitudes m can be plotted either against $\log a_1$, angular distances corresponding to a uniform exposure time, or against $\log E$, the brightness corresponding to a uniform distance; but with the scale properly chosen, the two figures would be identical, because of the relation derived from (3) and (4):

$$\log E_1 + \frac{2}{p}\log a_1 = \log E_0 + \frac{2}{p}\log a_0 = \text{Constant}$$

Log E_0 has been chosen as 60 minutes of time and $\log a_0$ as one minute of arc, and p has been assumed as unity. Thus,

$$\log a_1 = \log a + \tfrac{1}{2}(1.778 - \log E) \tag{5}$$

$$\log E_1 = \log E - 2 \log a \tag{6}$$

$$\log E_1 = 2 \log a_1 = 1.778 \tag{7}$$

Log a_1 was chosen as the quantity to be plotted against m in order that the result might be compared with Figure 1. The question of the proper value of p raises a serious theoretical difficulty which is minimized in the actual plotting by the use of a value for E_0 near the mean E, so that $\log E_0 - \log E$ in equation (5) is small. The various values of p used in discussions on stellar photometry range from about 0.75 to 1.00. No values have been directly determined for long exposures on nebular surfaces under actual observing conditions and there is very little basis on which to select the most probable value. However, the greatest difference in any particular $\log a_1$ as determined by equation (5) for the extreme values of p, is about 0.12. The mean difference is less than 0.04, which is of the same order of accuracy as the original data. These considerations seemed to justify the selection of $p = 1.00$ as the simplest procedure. The effect of using other values can be determined later, should the matter prove important.

Figure 2 shows the plot of $\log a_1$ against m. A least-squares solution gave for the adjusted curve

$$m+(4.90\pm0.13)\log a_1=11.02\pm0.10 \tag{8}$$

The simple mean of $m+5\log a$ is 11.10, and the corresponding curve differs from that of (8) by less than a tenth of a magnitude over the entire range of 14.2 magnitudes. The average deviation in m for both curves is about 0.8 magnitudes. The deviation of the coefficient of $\log a_1$ in (8) from the expected value of 5.0 is within the errors of observation.

A theoretical value can be computed for the constant on the right side of equation (8), by assuming the inverse-square law to hold rigorously and by assuming further that all the starlight intercepted by the nebulosity is re-emitted. Consider a point of nebulosity at distance a in minutes of arc from the star. The surface brightness of this nebulosity is determined on the above assumption by spreading the luminosity of the star over a spherical shell of radius a. Expressed in magnitudes per square second of arc, this surface brightness will be $m+2.5\log 4\pi(60a)^2$ or $m+11.64+5\log a$. Seares has determined the limiting surface brightness of an exposure of one minute on a Seed 30 plate, using a reflector of focal ratio 1 to 5.[1] This quantity, expressed in magnitudes per square second of arc, is 18.8 ± 0.3. It applies approximately to all photographic instruments of an equivalent focal ratio. For an exposure of sixty minutes corresponding to the values of $\log a_1$, the limiting surface brightness is $18.8+2.5\,p\times1.778$ or $18.8+4.45\,p$. Equating these two expressions,

$$\left.\begin{aligned} m+5\log a_1&=7.16+4.45\,p\\ &=11.61 \text{ for } p=1.00\end{aligned}\right\} \tag{9}$$

Assuming a random distribution of directions from star to nebulosity, the mean a observed, projected on a plane perpendicular to the line of sight, should be less than the true a by the factor $\pi/2$. This difference can be corrected by increasing m by $5\log\pi/2=0.98$. Equation (9) then becomes

$$\left.\begin{aligned} m+5\log a_1&=6.18+4.45\,p\\ &=10.63 \text{ for } p=1.00\end{aligned}\right\} \tag{10}$$

[1] *Mt. Wilson Contr.* No. 191; *Astrophysical Journal*, **52**, 162, 1920.

If the assumption that the light emitted by the nebulosity exactly equals the starlight intercepted is correct, then equation (10) should represent the points in Figure 2, and equation (9) should

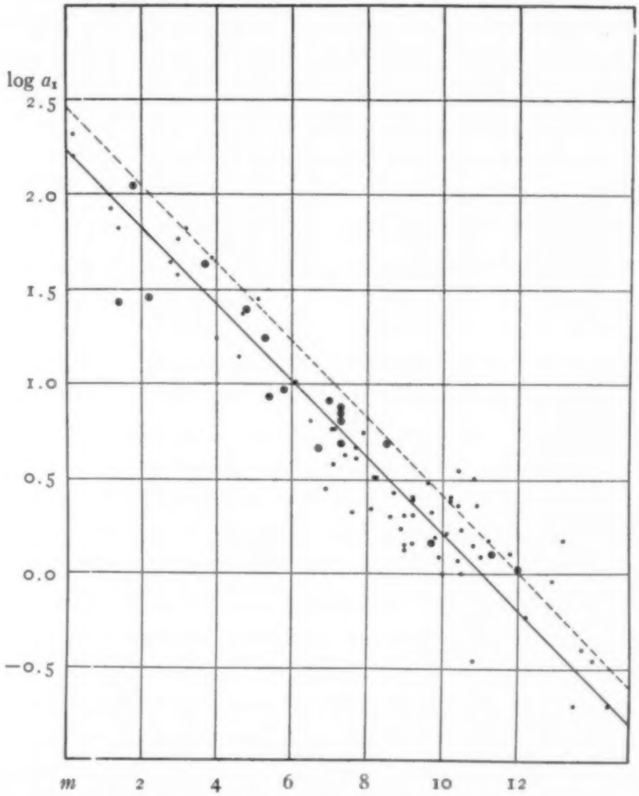


FIG. 2.—The same as Fig. 1, except that log a has been reduced to log a_1, corresponding to a uniform exposure time of one hour on Seed 30 plates with a reflector of focal ratio 1 to 5. The full line represents the curve $m+4.90 \log a_1=11.02$ derived from a least-squares solution. The broken line represents a limiting curve parallel to the full line, displaced 0.98 to the right.

play the rôle of an upper limit to the points. The agreement between (10) and (8) is close enough to raise a strong presumption that the assumption just mentioned is justified by the facts. Thus

(8) Observed. . . . . $m+4.90 \log a_1=11.02\pm 0.10$

(10) Theoretical. . . . . $m+5.00 \log a_1=10.63\pm 0.3$

The difference in slopes can scarcely be shown on the plot, and the difference in the constants on the right side of the equations is just the sum of the probable errors. It should be noted further that the constant in (8) falls nearly half-way between those for equations (9) and (10), the theoretical limiting curve and the mean curve on the assumption of random distribution of the directions from star to points measured for a. Any deviation from random distribution in the observed data would fall in this direction, for in each case the maximum a was consciously sought.

Observations and theory can also be compared by using $\log a$ and $\log E$ directly in a generalized form of equation (10). Assuming again that the total amount of starlight intercepted is re-emitted by the nebulosity

$$\left.\begin{aligned} 18.8\pm0.3+2.5\log E &= m+11.64+5\log a \\ m+5\log a-2.5\log E &= 7.16\pm0.3 \quad \text{(limit)} \\ &= 6.18\pm0.3 \quad \text{(mean)} \end{aligned}\right\} \qquad (11)$$

The left side can be computed from the original data and proves to be 6.65 ± 0.07 with an average residual of about 0.82. This lies almost exactly half-way between the theoretical limiting and mean values, and supports the suggestion that the data deviate from a random distribution in the direction of large angles to the line of sight. If the coefficient of $\log a$ in equation (11) be replaced by 4.90, as indicated in equation (8), the constant becomes 6.57.

This is not an independent method, for equations (9) and (10) reduce to equation (11) when the value of $\log a_1$ given in equation (5) is substituted into (9) and (10). The difference in the constant, $11.02-6.57=11.10-6.65=4.45$, is the quantity $2.5\,pE_0$ when p is taken as unity.

Equation (11) expresses a direct relation between the original data and should be the nicest method of procedure in the investigation. It is not well suited to least-squares solutions, however, because $\log a$ and $\log E$ are not independent. This difficulty is obviated by the transformation into equations (9) and (10).

Within the limits of error, equation (8) agrees with the theoretical mean curve expressed by equation (10). Disregarding for the

moment possible deviations from random distribution in the data, a limiting curve should appear on the plot parallel to the mean curve and displaced 0.98 magnitudes toward the side of increasing m. Any points falling to the right of this limit require explanation, for they indicate that the nebulosity is too bright or the stars too faint for a strict agreement with the theory that diffuse nebulosity re-emits exactly the light it receives from associated stars. Such a limiting curve has been sketched as a broken line on the plot in Figure 2, and several points do fall beyond. The general evidence is very strong against the nebulosity being too bright, i.e., that it emits more light than it intercepts from the star, and such an explanation should be appealed to only as a last resource.

The most probable solution appears to be the diminution of the stars' apparent brightness by nebulosity intervening between them and the earth. This is a well-known phenomenon among stars involved in nebulosity and makes itself felt in the abnormally large color indices which such stars as a general rule possess. The color excess, measured color index *minus* that normal to the spectral type of the star observed, is probably due to scattering of starlight by intervening nebulosity. If scattering is responsible for the color excess, then the true photographic magnitude of the star, as seen free from its screen of nebulosity, would be brighter than the observed value by about twice the color excess.

The nebulosity responsible for the observed scattering is obviously between the star and the earth. If the path from star to point of nebulosity measured for a were reasonably free from intervening nebulosity, such a correction should certainly be applied to the observed m. If, on the other hand, the nebulosity responsible for the scattering is distributed symmetrically about the stars, then the observed m is just that which is effective in illuminating the exterior points of nebulosity which were measured.

An examination of the points falling beyond the limiting curve indicates that as a rule they are measured from stars with large color excesses, and that a correction figured on this basis will throw them on the safe side of the curve. The stars from which these 13 points were measured, with D their distance in magnitudes

beyond the limiting curve and *CE* their measured color excesses, are listed below:

Star	*D*	*CE*
σ Scorpii	0.10	0.48
ρ Ophiuchi	0.15	0.50
B.D. +46°3474	1.05	0.97
B.D. +01177	1.20	1.00
B.D. −31013	0.10	0.22
B.D. −61417	0.20	0.55
B.D. −2312906	0.15	0.50
N.G.C. 1333	0.65	0.30
N.G.C. 2245	0.05	0.88
I.C. 446	0.30	0.80
N.G.C. 1624	0.10	....
N.G.C. 2185	0.70	....
Cometary Nebula	2.05	....

Color excesses are not available for the last three objects. N.G.C. 1624 is a small group of Oe5 stars involved in emission nebulosity. The combined magnitude is from estimated individual magnitudes and may be a few tenths in error. The stars in the last two nebulae show considerable color visually, and analogy with other objects of a similar nature suggests large values for the color excess. The cometary nebula is an uncatalogued object in a region of pronounced obscuration at $\alpha = 6^h3^m$, $\delta = +18°42'$ (1920) which shows a striking similarity to the variable nebula N.G.C. 2261. It is being followed for variability, but thus far without positive results. The color excess may not be large enough to account for the large *D*, but a small amount of general absorption could be postulated to make up the balance.

It is reasonable to suppose that in these 13 cases, the nebulosity responsible for the color excess is for the most part between the star and the observer, and that relatively little is between the star and point of nebulosity measured for *a*. An idea of how general is this dissymmetry of distribution about the stars of nebulosity responsible for the scattering can be derived from a study of relations between the residuals in Figure 2 and the color excesses. Measures of this last quantity are available for 63 points. There is no systematic difference in distribution between these and the points for which color excesses have not been measured. The

residuals may therefore be taken directly from Figure 2. A least-squares solution of the observational equations $x+y\,CE=$ Residual in m, gives

$$R\pm0.08=0.90\,CE-0.32 \tag{12}$$

If the scattering nebulosity were always symmetrically distributed about the stars, the coefficient of CE should be zero; if it were always between the star and observer and never between star and points measured for a, the coefficient should be approximately 2. An excess in scattering nebulosity between stars and points measured for a, over that between star and observer, would reduce the measured a and this effect could not be distinguished from that due to inclinations of the directions from star to measured points. The observed value of the coefficient of CE, namely 0.90, seems to indicate that the scattering nebulosity is distributed, not symmetrically, but rather at random about the stars. Corrections computed from equation (12) would provide only for the scattering nebulosity between star and observer in excess of that symmetrically distributed. It would reduce the constant in equation (8) by the quantity 0.32 and the resulting value, 10.70, would be in excellent agreement with that expected from theory, namely 10.63. Similar nebulosity not in the line of sight would reduce the true values of a, and a correction would in the mean tend to compensate the correction suggested by equation (12). The scattering effect probably will not seriously affect the mean values in Figure 2, although it offers a ready explanation for individual residuals.

A supplementary table of data was constructed from a homogeneous series of exposures made for the purpose of determining surface brightness of nebulosity from minimum exposure times necessary to show the nebulosity. The number of objects was small, but the results agree satisfactorily with those from Table I. The summation of $m+5\log a-2.5\log E$ is 6.50 ± 0.18, in good agreement with the theoretical value of 6.18 ± 0.30. The curve adjusted for the slope corresponding to the inverse-square law of luminosity is $m+5\log a_1=10.95\pm0.18$. The correlation curve is $m+4.38\log a_1=10.54\pm0.20$; but the points are not well distributed for a good determination of the slope, and the small value of the coefficient

of log a_1 cannot seriously affect the general conclusions. The plot of log a_1 against m is shown in Figure 3.

The data in Table II are entirely independent of those in Table I except the values for m. Different points were measured for a, and the exposure times average about one-sixth of those in Table I. The range in E for the two tables is especially significant, running in the first from 40 minutes to over 10 hours and in the second

TABLE II

Object	m	CE	Spec. Star	E	a	Log E	Log a	Log a_1	Instrument
				min.					
N.G.C. 1333	10.9	0.30	B9p	3.0	0'.15	0.477	−0.824	−0.174	60-inch
Merope *a*	4.0	0.00	B5	1.0	1.67	0.000	0.222	1.111	60-inch
Merope *b*	4.0	0.00	B5	0.25	0.40	−0.602	−0.398	0.792	60-inch
I.C. 348	9.0	0.80	B6–B9	4.0	0.32	0.602	−0.495	0.093	60-inch
359	13.5	0.00	K8d	50	0.17	1.699	−0.778	−0.738	100-inch
N.G.C. 1788	10.2	0.22	B8	1.0	0.17	0.000	−0.778	0.112	60-inch
I.C. 2118	0.2	0.00	B8p	30	135	1.597	2.130	2.220	10-inch
N.G.C. 2023	7.9	0.30	B2	0.33	0.33	−0.477	−0.477	0.651	60-inch
I.C. 435	8.2	0.20	B3	3.0	0.40	0.477	−0.398	0.252	60-inch
N.G.C. 2068	10.8	1.00	B5	0.50	0.40	−0.301	−0.398	0.641	60-inch
2071	10.8	0.80	B9	1.0	0.30	0.000	−0.523	0.376	60-inch
Orion	1.4	0.00	B0	30	210	1.597	2.322	2.412	10-inch
N.G.C. 2245	11.2	0.88	B1p	0.25	0.17	−0.602	−0.778	0.412	60-inch
2247	9.2	0.68	B2p	0.13	0.083	−0.875	−1.079	0.247	100-inch
I.C. 4592	3.9	......	B2	40	45	1.722	1.653	1.681	10-inch
4603	8.4	0.85	B2	30	10	1.597	1.000	1.090	10-inch
4604	5.1	0.50	B2–B3	20	25	1.421	1.398	1.577	10-inch
4605	4.7	0.12	B2	10	4.5	1.120	0.653	0.982	10-inch
N.G.C. 6726	7.2	0.10	B9	0.33	0.13	−0.477	−0.875	0.252	100-inch
6727	9.1	0.80	B9	0.60	0.13	−0.222	−0.875	0.125	100-inch
6729	10.0	0.50	Gp	0.25	0.10	−0.602	−1.000	0.190	100-inch
N.G.C. 7023	7.4	0.55	B2p	0.17	0.20	−0.778	−0.699	0.569	60-inch
7129	10.5	0.60	B3	0.33	0.072	−0.477	−1.176	−0.048	60-inch

The actual E is given for 10-inch plates, but 0.12 has been added to the value of log E to reduce to the Newtonian reflector system.

Merope *a* refers to the well-known wispy nebulosity, while *b* refers to the small bright cometary nebulosity about 30″ south of the star.

I.C. 359. The catalogue designation is uncertain. The position is given in the notes to Table I.

Orion refers to the brightest portion of the great exterior loop at about $\alpha = 5^h45^m.5$, $\delta = +1°$ (1920).

from 7 seconds to 50 minutes. Individual comparisons of log a_1 have no meaning, for inclinations of the directions from star to measured points cannot be determined independently. The systematic difference in log a_1 is 0.015, or about $3\frac{1}{2}$ per cent, which is satisfactorily small and indicates that the distribution of directions and of scattering nebulosity is comparable in the two systems of data.

The general conclusion from this investigation is that, within the errors of observation, the data can be represented on the hypothesis that diffuse nebulae derive their luminosity from involved or neighboring stars, and that they re-emit at each point exactly the amount

of light radiation which they receive from the stars. Where stars of sufficient brightness are lacking in the neighborhood, or, if present, are not properly situated to illuminate the nebula as seen from the earth, the clouds of material present themselves as dark luminosity.

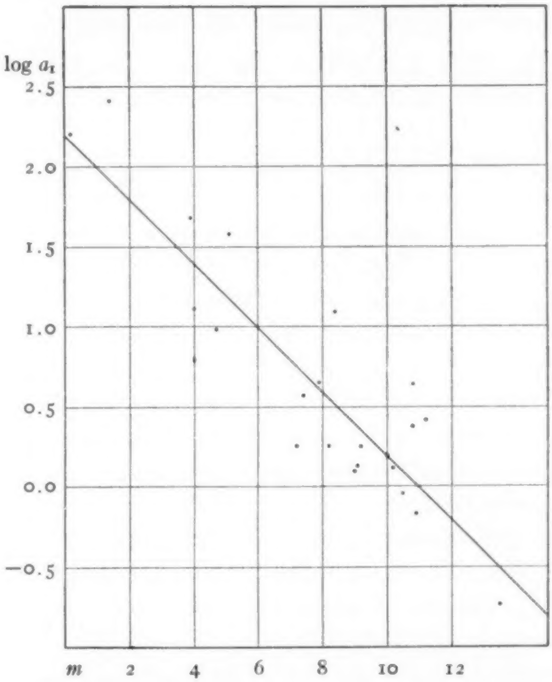


FIG. 3.—Photographic magnitudes of stars associated with diffuse nebulae (abscissae), plotted against the logarithm of the greatest angular extent of nebulosity from these stars (ordinates). The data are from Table II, derived from minimum exposures necessary just to register the brightest portions of the nebulae. Log a has been reduced to log a_1 corresponding to a uniform exposure of one hour on Seed 30 plates with a reflector of focal ratio 1 to 5. The straight line represents the equation $m+5 \log a_1 = 10.95$.

Eighteen of the objects listed in Table I show spectra which are predominantly emission. The points corresponding to these objects are mingled among the others in a thoroughly homogeneous manner and show no systematic characteristics whatsoever. Such a result was unexpected. The associated stars in these cases, with the exception of the Ob star B.D.$+37°3821$ in N.G.C. 6888, are of the

usual Oe5 and B0 types,[1] hence the nebular luminosity cannot be a phenomenon of simple reflection, but must result from some process of absorption and re-emission of stellar radiation. The data indicate that within the spectral limits to which Seed 30 plates are sensitive the continuous radiations from the stars are absorbed by the nebulosity and completely re-emitted as discontinuous radiations within the same range of wave-lengths. This may be a mere coincidence, but the idea is suggestive and should be investigated in detail.[2]

The fact that the emission nebulae do not shine by simple reflected starlight raises a question as to the process of illumination in the nebulae which show continuous or absorption spectra. The evidence given by V. M. Slipher as to the agreement between spectra of nebulae and of associated stars in the cases of Merope and Maia, ρ Ophiuchi, N.G.C. 2068, and N.G.C. 7023, is in favor of the reflection theory.

Analogy with emission nebulae, however, suggests the possibility that the process may be one of absorption and re-emission. This view is supported by the manner in which emission nebulae with no perceptible continuous spectrum merge smoothly into "continuous" with no emission as the spectral types of the dominating stars pass from Oe5 to the middle B's. It is extremely difficult, in the reflection theory, to account for the absence of strong continuous spectrum from purely emission nebulae. One must suppose that the physical state of this type of nebulosity is quite different from that of "continuous" nebulosity, being such that reflection is not possible. Further, it would lead to the logical conclusion that the physical state of nebulosity depends upon and is determined by the spectral type of the associated stars. Such a relation is not impossible, for the intensity of stellar radiation, closely associated with spectral type, must play a large rôle in determining the distribution of nebulous particles of various sizes, and thus indirectly bear upon the distribution of spectral characteristics throughout a given nebula. It seems impossible at present to strike a balance between

[1] γ Cassiopeiae and B.D. $-10°1848$ in I.C. 59, 63, and I.C. 2177 respectively show weak hydrogen emission lines.

[2] Plots similar to Fig. 2 were constructed for each of the various stellar spectral types in Table I; no systematic differences were found to exist.

the two views concerning the mechanism of illumination, but it is worth mentioning that the analogy of emission nebulae is quite as strong an argument against, as the agreement of spectra is in favor of, the theory of pure reflection.

Another striking characteristic of Figure 2 is the relatively small spread in the points and especially the complete absence of points falling far below the mean curve. The average deviation is about 0.8 in m or 0.16 in log a_1. The greatest single deviation to the left is 2.6 magnitudes or 0.53 in log a_1. A simple inspection of photographs shows at once that nebulosity as a rule is not symmetrically distributed about stars. Nevertheless Figure 2 indicates that as a rule nebulosity is found at the extreme limits to which a star will illuminate it sufficiently to be photographed with exposures of several hours duration. The explanation is probably to be found in relations between pressure and illumination effects of the stellar radiations on nebulous material of the sizes responsible for the nebular luminosity.

Several of the objects in Table I are of considerable interest individually. γ Cygni for instance is a pseudo-Cepheid with a spectral type F8p. The radiative symmetry of the several neighboring patches of nebulosity raises a strong presumption that they are associated with the star, and this is supported by the positions of the corresponding points in Figure 2. The same can be said concerning α Cygni and the North America nebula, N.G.C. 7000, as well as β Orionis and the nebula I.C. 2118 with its long extension into Eridanus. N.G.C. 1514 is really a giant planetary, conspicuous for the exceptionally bright central star.

One well-known galactic diffuse nebula is a conspicuous exception. The only object investigated for which no obviously dominating star could be located and for which no pronounced obscuration was conveniently situated to hide such a star, is the great loop in Cygnus of which N.G.C. 6960 and 6992 are brighter portions. These portions give emission spectra as could be expected from their filamentary structure. They are opposite one another at the ends of the longer diameter of the loop. Barnard's photograph reproduced on Plate 80 of the *Lick Publications*, Volume XI, shows the configuration. The major axis is roughly $2^{\circ}5$. One would expect

to find a star of spectral type B0 or earlier approximately central, and hence about a degree and a quarter from the outer edges of the nebula. From Figure 2, the apparent photographic magnitude should be around 2.0, with a probable error of less than a magnitude. In the immediate vicinity no early-type stars can be found as bright as 9.0, the limit to which the search was pushed. The evidence from the other diffuse nebulae is too strong to admit the Cygnus loop as a real exception, shining by its own intrinsic luminosity, and explanation must be sought wherever possible. The obvious assumption is that there is a small cloud of dark nebulosity immediately in front of a central star, sufficiently opaque to dim the star by several magnitudes but inconspicuous on small-scale photographs.

Photographs of the central region made with the 100-inch reflector show no certain traces of such dark areas, and general obscuration cannot be present for at least one spiral nebula is shown on the plates. There is considerable filamentous nebulosity in this region, of much the same structural characteristics as the bordering patches of nebulosity known as N.G.C. 6960 and 6992. This suggests that the nebula as a whole may be an ellipsoidal shell rather than a simple loop. B.D. $+30^\circ 4199$, the brightest star within the loop, which is not far from the center, has enough nebulosity immediately around it to be called a nebulous star. Its visual magnitude is given in the B.D. as 7.0, and its spectrum is of highly enhanced type similar to that of α Cygni. It is just possible that this star, dimmed several magnitudes by general absorption of intervening nebulosity, is the source of illumination of the loop. This would be analogous to the association of α Cygni itself and the similar star B.D. $-22^\circ 4510$ with emission nebulosity.

Another point worth mentioning is that, using the observed magnitude for B.D. $+30^\circ 4199$, the relation between m and a for the loop would be of the same order as those for the Crab nebula (N.G.C. 1952), the Dumbbell nebula (N.G.C. 6853), and the Helical nebula in Aquarius (N.G.C. 7293). These objects appear to form a separate group, and speculation concerning their exceptional nature may well be deferred until the luminosity of planetaries has been investigated in a quantitative manner.

An interesting application of the conclusions arrived at in this paper can be made in the case of the spiral M 33. The bright knot in the spiral known as N.G.C. 604 has an emission spectrum of the galactic diffuse nebular type. That is to say, hydrogen is so strong as compared to nebulium that $H\beta$ is of about the same intensity as N_1. This is a general rule among diffuse emission nebulae, but is exceptional among the planetaries. Three of four stars involved in N.G.C. 604 strengthen the analogy with galactic diffuse nebulae. If we assume the same conditions to hold in N.G.C. 604 as in the galactic nebulae, these stars must be considered as probably Bo or Oe5, and Kapteyn's value of -2.5 for the mean absolute magnitude of Bo stars can be used with some degree of justification to estimate the parallax. The apparent magnitudes of these stars have not been measured, but a casual inspection of plates made with various instruments and exposures suggests that the order of magnitude of the brightest is about 15. An exposure of two hours shows nebulosity extending to about $10''$ from the brightest star, so the corresponding point in Figure 2 would fall very close to the mean curve, and the analogy with galactic nebulae is still further strengthened. Since the brightest star alone is used, Kapteyn's value for the mean absolute magnitude can scarcely be considered too bright. The resulting modulus, $m-M$, of 17.5 corresponds to a parallax of $0\overset{''}{.}000032$ or a distance of about 30,000 parsecs. If absorption were present, dimming the apparent brightness of the star, the distance would be correspondingly decreased. Measures of the color index would probably determine this point.

The general equations (11) expressing the relations between m, a, and E can be transformed into corresponding relations between M, the absolute magnitude (photographic in this case) and l, the greatest linear distance in parsecs to which luminous nebulosity extends, by means of

$$m=M-5-5\log\pi$$
$$a=3438\,\pi\,l$$

The expression for a is approximate, the exact relation being $l=2/\pi\sin a/2$, but the difference within the range covered by a is

practically negligible. Substituting into the limiting general equation (11),

$$M+5 \log l=2.5 \log E-5.52 \tag{12}$$

It follows that for exposures of about 160 minutes on Seed 30 plates with reflectors of focal ratios 1 to 5, representing therefore a limiting surface brightness of $18.8+2.5 \log 160=24.32$ magnitudes per square second of arc, the simple relation holds,

$$M+5 \log l=c \tag{13}$$

This will represent average conditions of direct nebular photography. A short table of corresponding values will emphasize the enormous distances involved.

	Parsecs		Light Years
For $M=-5$	$l=10$	or	32
-2.5	3.2		10
0	1.0		3
$+2.5$	0.31		1
5	0.1		0.3
10	0.01		0.03

The sun, with a photographic absolute magnitude around 5.65, would be capable of illuminating a surface to a brightness of 24.32 magnitudes per square second of arc at a distance of 0.074 parsecs = 0.24 light years $=1.5\times 10^4$ astronomical units, or 470 times the mean distance of Neptune.

The greatest value of l encountered among the nebulae is for the nebulosity preceding Rigel. The absolute photographic magnitude of the star is about -5.58.[1] An exposure of ten hours with a 1.5-inch Tessar Ic lens of focal ratio 1 to 4.5 corresponds roughly to an exposure of thirteen hours with the reflectors, and from equation (13) the value of l should be about 29 parsecs or 93 light years. On the plate, nebulosity can be traced south preceding Rigel to a distance of nearly $12^\circ\!.5$. Using Kapteyn's value of $0''\!.0069$ for the parallax of Rigel, this corresponds to a distance of nearly 31 parsecs or a hundred light years.

[1] Kapteyn's value of -5.5 for the visual absolute magnitude is corrected for the normal color index of a B8 star.

II. PLANETARIES

The fidelity with which the luminosity of diffuse nebulae having emission spectra obeys the laws derived from data obtained mainly from nebulae with continuous spectra, suggests the possibility of applying the methods developed in the preceding discussion to an investigation of luminosity relations in the planetaries. Spectra of the nuclei or central stars of these objects are predominantly continuous. Emission bands are frequently present, but, except for three or four objects, the energy in these bands is a negligible fraction of the energy in the complete spectrum. The law is definitely established that the spectra of nucleus and of nebulosity are dissimilar. In these respects the spectral relations between nebulosity and central stars are analogous to those in the case of the diffuse nebulae having emission spectra.

In other respects, however, the analogy fails to hold. The emission lines N_1 and N_2 are as a rule much stronger in the planetaries than in the diffuse nebulae. Since they are situated very near the red limit of sensitivity of Seed 30 plates, the tendency is to destroy the similarity of intensity distribution between stellar and nebular spectra which appears to hold for diffuse emission nebulae and which permits these objects to obey the same luminosity laws as diffuse nebulae with continuous spectra. Moreover, the extraordinary intensity in the ultra-violet region of the spectra of planetary nuclei introduces a new element into the luminosity relations which may differentiate them from those found among diffuse nebulae.

Difficulties are to be expected in an investigation of luminosity relations between stars with predominantly continuous spectra and nebulosity with complicated emission spectra, and, in the absence of definite theories of the mechanism of nebular illumination, the results will be in the nature of general tendencies rather than precise laws. Provisional results, however, should at least indicate some of the special features to be considered in a later and more thorough investigation.

Curtis'[1] descriptive catalogue of planetaries visible in the northern skies furnishes fairly homogeneous data for 78 objects. For

[1] *Lick Observatory Publications*, **13**, Part III, 1918.

each nebula he gives an illustration, an estimate of the photographic magnitude of the central star, the measured dimensions of the nebulosity, and estimates of its brightness expressed by the relative exposure necessary to register the brightest nebular feature on Seed 27 plates at the primary focus of the Crossley reflector.

In Table III are collected the data for the 56 objects in Curtis' catalogue which have more or less conspicuous central stars. The first column gives the designation; the second, the estimated magnitude m of the central star; the third, the angular distance, A, in seconds of arc measured from the central star to the brightest details of the nebulosity, which, as well as could be determined, are those details for which the relative exposure times are given by Curtis; the fourth, the relative exposures, e; the fifth and sixth columns, the logarithms of A and e, respectively; the seventh, the quantity $\log A_1$, the significance of which will be explained later. The values of m and e are those given by Curtis. When A is given by Curtis, his value is listed in Table III; otherwise it is estimated from the illustrations accompanying the catalogue.

An inspection of the table indicates no trace of correlation between m and $\log A$, but considerable, though vague, evidence of a relation between m and $\log e$. The most conspicuous feature, however, is a rather definite relation between m and a combination of $\log A$ and $\log e$, in the sense that bright central stars are usually associated either with bright and small or with faint and very large nebulae. Furthermore, a study of the illustrations establishes the fact that, where multiple rings are present in a nebula, the order of brightness is usually inversely that of the radii or distances from the central star, and that, in the structureless disks, there is a tendency for the brightness to diminish outward from the central star. These facts, together with the conclusions derived from the discussion of diffuse nebulae, suggest a reduction of $\log A$ to $\log A_1$ corresponding to a uniform relative exposure, on the assumption that the central stars are the sources of luminosity and that, for a homogeneous distribution of nebulosity, the inverse-square law is rigorously obeyed.

TABLE III

PLANETARY NEBULAE

Object	m	A	e	log A	log e	log A_1
N.G.C. 40	10	17″	4	1.23	0.60	0.93
246	9.5	100	100	2.00	2.00	1.00
650, 1	16	40	20	1.60	1.30	0.95
I.C. 1747	14	6	15	0.78	1.18	0.19
351	14	4	2	0.60	0.30	0.45
N.G.C. 1501	12	24	25	1.38	1.40	0.68
1514	8.8	60	80	1.78	1.90	0.83
1535	10	10	2	1.00	0.30	0.85
J 320	12	3.5	0.8	0.54	−0.10	0.59
I.C. 418	9	7	0.2	0.85	−0.70	1.20
N.G.C. 1952	15.2	50	60	1.70	1.78	0.81
2022	13	11	5	1.04	0.70	0.69
I.C. 2149	12	3	0.3	0.48	−0.52	0.74
N.G.C. 2371, 2	12	17	15	1.23	1.18	0.64
2392	9	9	5	0.95	0.70	0.60
2438	16	30	50	1.48	1.70	0.63
2452	19	8	20	0.90	1.30	0.25
2610	15	18	60	1.26	1.78	0.37
3242	9	13	3	1.11	0.48	0.87
3587	12	100	100	2.00	2.00	1.00
4361	10	20	30	1.30	1.48	0.56
I.C. 3568	11	5	0.2	0.70	−0.70	1.05
N.G.C. 6058	12	10	30	1.00	1.48	0.26
I.C. 4593	10	5	2	0.70	0.30	0.55
N.G.C. 6210	11	4	0.3	0.60	−0.52	0.86
6309	13	5	4	0.70	0.60	0.40
6369	16	14	70	1.15	1.85	0.23
6439	18	2	8	0.30	0.90	−0.15
6445	19	18	20	1.26	1.30	0.61
6543	9.2	8	0.2	0.90	−0.70	1.25
6563	18	18	60	1.26	1.78	0.37
6572	9	3	0.1	0.48	−1.00	0.98
6567	14	3	0.6	0.48	−0.22	0.59
6578	15	4	8	0.60	0.90	0.15
6620	15	2.5	4	0.40	0.60	0.10
6629	13	7	10	0.85	1.00	0.35
6720	13	30	6	1.48	0.78	1.09
6751	12	10	20	1.00	1.30	0.35
6772	18	26	250	1.41	2.40	0.21
6778	14	9	15	0.95	1.18	0.36
6781	14	50	80	1.70	1.90	0.75
6803	13	2.7	0.3	0.43	−0.52	0.69
6804	12	20	25	1.30	1.40	0.60
6818	14	10	0.7	1.00	−0.15	1.07
6826	9	11	1	1.04	0.00	1.04
6853	12	90	20	1.95	1.30	1.30
6891	10	3.5	2	0.54	0.30	0.39
6894	16	22	70	1.34	1.85	0.42
6905	13	18	20	1.26	1.30	0.61

TABLE III—*Continued*

Object	m	A	e	log A	log e	log A_1
N.G.C. 7008	12	40	20	1.60	1.30	0.95
7009	11	12	0.2	1.08	−0.70	1.43
7026	14	3	0.8	0.48	−0.10	0.53
7139	18	35	300	1.54	2.48	0.30
7293	11	300	150	2.48	2.18	1.39
7354	16	8	15	0.90	1.18	0.31
7662	11.5	8	0.2	0.90	−0.70	1.25

These assumptions lead to a reduction formula analogous to equation (3). Let the uniform relative exposure be unity and let the exponent p also be taken as unity. Then

$$\log A_1 = \log A - \tfrac{1}{2} \log e \qquad (14)$$

Log A_1 is listed in the seventh column of Table III.

Figure 4 shows the plot of m against log A_1 for the 56 planetaries in Table III. Some degree of correlation is obvious, but the coefficient is so small that a least-squares solution in the form used in the investigation of diffuse nebulae does not give satisfactory results. Accordingly the data were treated by the method of correlations. The two regression curves are

$$RC \text{ of } m \text{ on } \log A_1 \; . \; . \; . \; . \; m + 4.63 \log A_1 = 16.03 \qquad (15)$$

$$RC \text{ of } \log A_1 \text{ on } m \; . \; . \; . \; . \; m + 13.35 \log A_1 = 21.84 \qquad (16)$$

Coefficient of correlation 0.59

$$\sigma_m = 2.80 \qquad \sigma_{\log A_1} = 0.356$$

The correlation ratio of equation (15) is 0.61, and of (16), 0.64. Attention should be called to the fact that the regression curve of m on log A_1, determined by plotting the mean m for groups of log A_1, is not a straight line but a rather well-defined curve convex to the origin of co-ordinates and hyperbolic in form. The correlation ratio is computed from this curve. The reason for such a curve is to be found in the distribution of errors, especially the systematic errors in m and e, which will be discussed later. Equa-

tion (15) is the straight line which best fits the points of the regression curve. The regression curve of log A_1 on m is a straight line.

The best simple expression for the relation between m and log A_1 is probably the curve of symmetry, or the bisector of the acute angle between the lines represented by equations (15) and (16). This

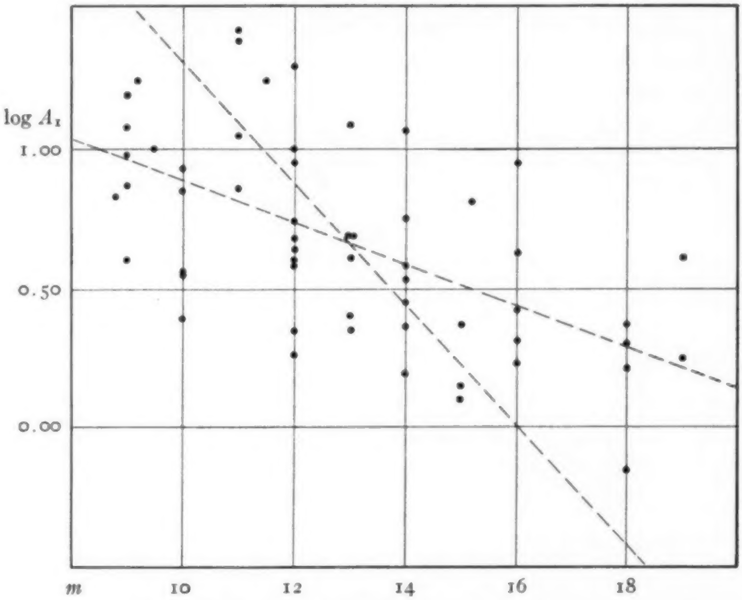


FIG. 4.—Plots of m, the photographic magnitude of central stars in planetary nebulae, against log A_1, where A_1 is the angular extent of nebulosity from the central stars reduced to a uniform exposure time. The data are taken from Table III. The broken lines are the regression curves.

reduces the sum of the perpendicular distances from the points to the line to a minimum. The bisector is

$$m+(6.91\pm0.6)\log A_1=17.55\pm0.22 \qquad (17)$$

The data are not of a high order of accuracy but the mean values should be significant. The distances A must be essentially free from serious systematic errors, although the accidental errors are probably large as compared to the values of A, especially in the many very small nebulae.

The relative exposure, e, seems to be fairly reliable for large and faint nebulae, but is probably too large for the small and bright objects as compared with larger-scale photographs made with the 60-inch and 100-inch reflectors. This may be due to the known dependency of minimum visibility on surface area, and to the difficulty on small-scale photographs of distinguishing the star image from bright nebulosity in its immediate neighborhood. Such errors would be especially serious in the case of stars involved in very small dense nebulae.

TABLE IV

COMPARISON OF MOUNT WILSON LIMITING EXPOSURE TIMES WITH CURTIS' RELATIVE EXPOSURES

Object	E	e	$R=\frac{E}{e}$
	a) Large and Faint Nebulae		
	sec.		
N.G.C. 246	600	100	6.00
1514	300	80	3.75
3587	600	100	6.00
6445	150	20	7.50
6853	120	20	6.00
6894	480	70	6.86
7293	900	150	6.00
Mean			6.02±0.28
	b) Small and Bright Nebulae		
	sec.		
N.G.C. 3242	1	0.3	3.33
I.C. 3568	4	2.0	2.00
4593	6	2.0	3.00
N.G.C. 6210	1.5	0.3	5.00
6572	0.4	0.1	4.00
6720	10	6	1.67
7009	0.8	0.2	4.00
7662	0.8	0.2	4.00
Mean			3.38±0.33

Minimum exposures in seconds of time required just to register the brightest details of nebulosity on Seed 30 plates at the primary focus of the 60-inch reflector have been derived by the writer for 15 planetaries. These are compared with Curtis' values of e in Table IV. The first column designates the nebulae, the second gives

the minimum exposures E in seconds of time derived from the Mount Wilson plates, the third gives Curtis' values of e, and the fourth contains the ratios $R=E/e$ which represents the minimum exposure, expressed in seconds, made on Seed 30 plates with the 60-inch reflector, corresponding to Curtis' unit of relative exposure.

The values of e for N.G.C. 3242 and I.C. 3568 have been arbitrarily changed by shifting the decimal points one place to the left and to the right, respectively. Curtis' value of 3 for N.G.C. 3242 is unquestionably a misprint for 0.3, since he remarks that the inner ring shows in 10 seconds on a Seed 23 plate. His description of I.C. 3568 places its brightness about equal to that of I.C. 4593. The value $e=6$ for N.G.C. 6720, the ring nebula in Lyra, is surprisingly high. The nebula may be photographed in 10 seconds with the 60-inch, and a rather strong image registers in 15 seconds.

The unit of relative exposure corresponds to an exposure of 10 seconds on a Seed 27 plate with the Crossley reflector. A standard region in the Orion nebula near the trapezium shows well under these conditions.[1] When one considers the more favorable focal ratio of the 60-inch reflector, and the difference in intensities between that "just perceptible" and that "well shown," the ratio $E/e=6.0$ found for the large faint nebulae seems to be a reasonable value. The relative speeds of the two instruments, allowing for the fact that the Crossley plates are made at the direct focus without the use of a Newtonian flat, is about 1.18 in favor of the 60-inch. The factor due to the different standards of intensities used in the two series is a matter of conjecture and can be variously estimated at from 1.25 to 1.50, in the sense that the smaller intensities, and hence shorter exposures, are employed by the writer. Using a mean value of 1.38 for this factor, and considering the difference in speed between Seed 27 and Seed 30 plates to be negligible, the computed ratio E/e should be about $10\div1.18\times1.38=6.14$. This agrees with the observed value, 6.0, for the large faint nebulae and indicates that the discrepancy between the ratios of the two lists in the preceding table is due to errors in the small bright objects.

The evidence seems rather definitely in favor of a systematic error in e for the very bright nebulae, but the data are not sufficient

[1] Curtis, *op. cit.*, p. 59.

to determine the limitations of the error. A correction would decrease e for these objects and hence increase log A_1. Since the bright nebulae as a rule have bright central stars, the effect would be to raise the upper ends of the correlation curves compared to the lower ends, and to decrease the value of the coefficient of log A_1 in equation (17). It is probable that the systematic error in e is not serious for values of e greater than 2.0. The effect on equation (17) can be estimated by applying to each e a uniform correction equal to 2 or less and considering any errors in the larger e's as accidental. As there are only four objects with e greater than 2 and less than 5, beyond which the systematic error is almost certainly negligible, this course seems justifiable. The correction to be applied is computed from the mean of the values for the bright nebulae in Table IV, disregarding N.G.C. 6720, the discrepancy in which may be considered as an accidental error. Each e equal to 2 or less must then be divided by $6.2 \div 3.62 = 1.72$. Log e is accordingly decreased by 0.24, and log A_1 increased by 0.12. This correction is applied to eighteen objects. The mean log A_1 of Table III, 0.667, will be increased by $0.12 \times 18/56 = 0.039$. The corrected mean value will be 0.706. The mean m will remain the same, hence the corrected curve must pass through the point $m = 12.95$, log $A_1 = 0.706$. For the objects in which e has been corrected, the mean m is low, 11.3, and no m is greater than 14. Likewise the mean log A_1 is high, 0.86, and only two are less than 0.50. The result is that the intersection of the curve with the m-axis at $m = 17.55$ will be very little disturbed. The corrected curve will pass close to this point and through the point $m = 12.95$, log $A_1 = 0.706$. The equation of the corrected curve will therefore be approximately

$$m + 6.52 \log A = 17.55 \qquad (18)$$

Curtis' magnitudes are estimates and therefore subject to rather large accidental errors. In addition, there is a pronounced systematic error for the brighter stars, in the sense that they are estimated too bright.

Van Maanen has given photographic magnitudes of the central stars of the planetaries which he has measured for parallax.[1] The

[1] *Mt. Wilson Contr.* No. 237, 1922.

method he employs is to count the numbers of stars equal to or brighter than the ones in question on his parallax plates and to extract the corresponding magnitude from van Rhijn's[1] tables of average stellar densities for given galactic latitudes. This procedure is subject to considerable accidental error in individual cases, especially of the brighter stars, for the field used is small enough to be seriously affected by local irregularities in stellar distribution. In the mean, however, they should be reliable within the limits of Curtis' accidental errors.

The writer has derived provisional photographic magnitudes for several central stars, using the method of polar comparisons, with exposures timed to show a minimum of nebulosity. In general each of the values depends upon a single plate and they are not, therefore, very reliable. They should, however, be of the same order of accuracy as van Maanen's determination, and both series can be used to reduce Curtis' measures to the scale of the polar sequence. The data are given in Table V. Van Maanen's value of 8.0 for the central star in N.G.C. 1514 is omitted for the reason that the star is too bright for a good determination of the magnitude by the method of star counts.

The mean difference, van Maanen − Curtis, is 1.3, corresponding to a mean magnitude (Curtis) of 11.5. The mean, Hubble − Curtis, is 1.4 for a mean magnitude (Curtis) of 12.0. Between Curtis' magnitudes and the differences vM − C and H − C, called Δ, the combined data give a coefficient of correlation of 0.44, which has very little significance. The mean point of the material is at m (Curtis) $= 11.7$, $\Delta = +1.35$. This probably represents a systematic error in Curtis' values amounting to -1.35 at his magnitude 11.7. It is improbable that the error should be uniform over the range of m covered by Table III. A more reasonable assumption is that the error vanishes near the limiting magnitude obtainable with the Crossley in an exposure of moderate length. This supposition is supported by the two faint central stars in Table V for which Δ is negligible. If this is the case, the correction to Curtis should be represented approximately by a straight line drawn through the mean point of the data of Table V, rejecting

[1] *Groningen Publications*, No. 27, Table IV, 1917.

the two faint stars in N.G.C. 6445 and 6772, and cutting the axis of m at an arbitrarily chosen point, say m equals 19.5. The formula expressing Δ, the correction to be added to Curtis' m, is then

$$\Delta = -0.173\, m + 3.37 \tag{19}$$

The effect of this assumed correction would be in the nature of a rotation of the correlation curves of Figure 4 in a clockwise direction

TABLE V

PHOTOGRAPHIC MAGNITUDES OF PLANETARY NUCLEI

Object	Curtis	van Maanen	Hubble	vM−C	H−C	H−vM
N.G.C. 40	10	11.6	....	1.6	....	....
1501	12	13.0	....	1.0	....	....
1514	8.8	....	9.7	....	0.9	....
2022	13	14.2	....	1.2	....	....
2371, 2	12	13.5	....	1.5	....	....
2392	9	10.0	....	1.0	....	....
3242	9	....	11.7	....	2.7	....
3587	12	....	14.3	....	2.3	....
I.C. 3568	11	....	12.0	....	1.0	....
4593	10	....	10.2	....	0.2	....
N.G.C. 4361	10	....	12.8	....	2.8	....
6210	11	11.7	....	0.7	....	....
6445	19	....	19	....	0.0	....
6543	9.2	11.3	....	2.1	....	....
6572	9	10.8	10.2	1.8	1.2	−0.6
6720	13	14.7	14.7	1.7	1.7	0.0
6772	18	....	18.1	....	0.1	....
6804	12	13.4	....	1.4	....	....
6853	12	....	13.6	....	1.6	....
6905	13	14.5	....	1.5	....	....
7008	12	12.8	....	0.8	....	....
7009	11	....	11.7	....	0.7	....
7026	14	15.1	....	1.1	....	....
7293	11	....	13.3	....	2.3	....
7662	11.5	12.9	12.5	1.4	1.0	−0.4
Mean	11.7	....	....	1.34	1.36	....

around the points where they cross the abscissa $m = 19.5$. Equation (18) corrected in this manner becomes

$$m + 5.40 \log A_1 = 17.88 \tag{20}$$

This curve is probably a fair representation of the data, disregarding the probable misprints in the decimal points for e in the cases of N.G.C. 3242 and I.C. 3568. Taking these into account

and correcting Curtis' values of m and e for the systematic errors suggested by the Mount Wilson observations, the recomputed correlation curves are:

RC of m on $\log A_1$ .... $m+3.71 \log A_1=16.73$

RC of $\log A_1$ on m .... $m+9.21 \log A_1=20.70$

Correlation curve .... $m+(5.33\pm 0.4) \log A_1=17.88\pm 0.18$ (21)

Coefficient of correlation 0.64

$$\sigma_m=2.28 \qquad \sigma_{\log A_1}=0.389$$

The plot corresponding to the corrected data is shown in Figure 5.

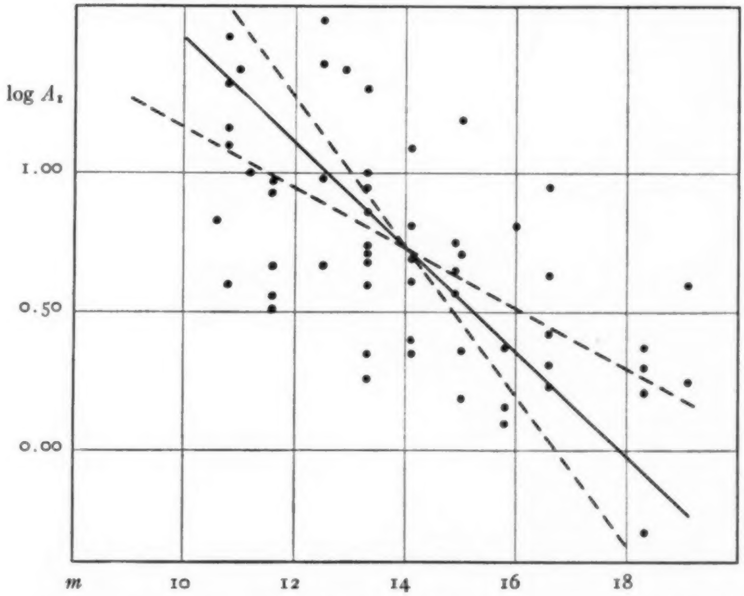


FIG. 5.—The same as Fig. 4, the data having been corrected for probable systematic errors. The full line represents the correlation curve for an equal weighting of the regression curves.

A curve passing through the mean m and $\log A_1$ and having the slope to be expected on the assumption of an inverse-square law of luminosity can be represented by the equation

$$m+5 \log A_1=17.64\pm 0.17 \qquad (22)$$

The two equations (21) and (22) agree within the limits of errors over the range covered by the data.

The general conclusion is in favor of the theory that the planetaries as well as the diffuse nebulae derive their luminosity from radiations of associated stars, and that the inverse-square law is at least one important factor in determining the distribution of luminosity throughout the nebulae. The complicated discontinuous nebular spectra of the planetaries, the great intensity in the ultraviolet of the continuous nuclear spectra, and the obvious concentration of luminous nebulosity in restricted zones, probably disturb the simple application of the inverse-square law sufficiently to account for the large residuals found in the present investigation.

There are some indications that the residuals computed from equations (21) and (22) may be related to the various types of planetaries. The four ring nebulae, N.G.C. 2438, 6720, 6894, and 7293, all show large positive residuals, averaging $+1.80$, while the globular objects I.C. 3568, 4593 and N.G.C. 6439, 6572, 6578, 6620, and 6629 all show large negative residuals, averaging -1.50. It is possible that when more precise data are available, a complete sequence can be established involving relations between residuals, on the one hand, and, on the other, spectral characteristics of nuclei and of nebulosity, color indices of nuclei, and structure of nebulosity.

It is worth mentioning at this point that when referred to B.D. $+30°4199$ as the central star, the loop in Cygnus discussed in the first part of this paper takes its place among the ring nebulae. The maximum distance A, measured to the following edge of N.G.C. 6992, which registers in an exposure of forty minutes, is $90'$. $m+5\log A_1=7.2+12.0=19.2$. The residual, $19.2-17.6=+1.6$, is of the order of that for the other ring nebulae.

Assuming the inverse-square law to hold rigorously with respect to the central stars as sources in the case of the planetaries, and assuming further that, within the range of wave-lengths to which the fast Seed plates are sensitive, the light emitted by the nebulosity is exactly that which it intercepts from the stars, a theoretical relation can be derived analogous to that in equation (9). Assumptions of random distribution in direction from stars to points of nebulosity measured for A are not necessary, for the symmetrical forms of the

planetaries indicate that in general the directions of greatest extent of nebulosity from the central stars lie close to the plane perpendicular to the line of sight. The surface brightness of nebulosity on the above assumptions, expressed in magnitudes per square second of arc, is found by distributing the light of the star over a spherical shell with a radius equal to the distance to the point of nebulosity considered. Let this distance, A, be expressed in seconds of arc. Then

$$S.\,B. = m + 2.5 \log 4\pi A^2 = m + 5 \log A + 2.75 \tag{23}$$

The unit of relative exposure corresponds to an exposure of six seconds with the 60-inch reflector, or, if Seares's value for the limiting surface brightness just registered in an exposure of one minute is used[1] to a limiting surface brightness of $18.8 \pm 0.3 + 2.5 \log 0.1 = 16.3 \pm 0.3$. Substituting this value for $S.B.$ into equation (23),

$$m = 5 \log A = 13.55 \pm 0.30 \tag{24}$$

which expresses the relation between m and $\log A$, where A is measured from the star to the most distant nebulosity which just shows in an exposure of six seconds.

A comparison between this equation and the observational equations (21) and (22) shows a discrepancy in the constants amounting to from 4.1 to 4.3 ± 0.6 magnitudes, in the sense that in general the observed nebulosity extends to a greater distance from stars of a given magnitude than the assumptions permit. Expressed in another way, the nebulosity at a given distance from the star is about 4.1 to 4.3 ± 0.6 magnitudes per square second of arc brighter than can be accounted for on the assumption that, within the range of the fast Seed plates, the light emitted by the nebulosity is exactly the starlight intercepted. This may well account for the apparent absence of central stars or nuclei from some of the planetaries.

No adequate explanation of this discrepancy can be made without a knowledge of the mechanism by which the nebular illumination is excited, but a few suggestions may aid in formulating the problem. The source of illumination is rather definitely established

as located in the central stars. The diffuse nebulae with emission spectra fail to show this discrepancy although it seems reasonable to suppose that the mechanism of illumination is much the same. The most striking difference between the nebular spectra of the planetaries and the emission spectra of diffuse nebulae is found in the much greater strength of the nebulium lines N_1 and N_2 in the planetaries. These lines are situated very close to the limits of sensitivity toward the red of the Seed 30 plates, and thus tend to destroy that equivalence of intensity distribution which the spectra of emission diffuse nebulae appear to share with the spectra of their associated stars, and by virtue of which they actually follow the same luminosity laws as do the diffuse nebulae with continuous spectra. The strengthening of the nebulium lines in the planetaries is accompanied by no corresponding strengthening of the continuous spectrum of the nuclei in this region. The maximum intensity of the continuous nuclear spectrum is shifted to the violet as compared with that of the stars associated with diffuse nebulae, and this in effect decreases the relative intensity in the region of the nebulium lines. If the nebular emission lines were considered as integrated from the continuous spectrum in their immediate vicinity, then the strong nebulium lines would represent a considerable part of the green region to which the Seed 30 plate is relatively insensitive. Such a supposition, if it were physically possible, could account for only a small part of the discrepancy, for these two lines seldom furnish more than a third of the nebular luminosity, as is shown by Wright's measures of relative intensities of nebular images made on spectrograms obtained with a slitless quartz spectrograph and a reflecting telescope, which are comparable with the direct photographs used in deriving the data of the present discussion.[1]

Another difference between planetaries and diffuse nebulae is found in the greater intensity of the ultra-violet continuous spectrum of the planetary nuclei as compared to that of the stars associated with diffuse nebulosity. The nuclear spectra are in general predominantly continuous. When corrected for atmospheric absorption, the energy curve of the continuous spectrum rises more and more steeply with decreasing wave-length up to the limits of reflec-

[1] *Lick Observatory Publications*, **13**, Part VI, 1918.

tivity of silvered mirrors. Wright remarks, "Where the maximum is cannot be conjectured." It is certainly at a wave-length less than 3300 A. Thus the nebulosity receives a vast amount of energy in stellar radiations in a region of the spectrum which plays but a minor rôle in the determination of photographic magnitudes. It is entirely conceivable that the energy in these continuous radiations may be absorbed by the nebulosity and, by some mechanism analogous to that of fluorescence, be re-emitted as discontinuous radiations of longer wave-lengths. If this were the case, the m in equation (20) would be replaced by one of smaller numerical value representing a sort of ultra-violet magnitude, and the discrepancy in the constants would be materially reduced, if not entirely eliminated. Following this line of thought, the suggestion arises that the nebulium lines may possibly reveal themselves as fluorescent spectra of hydrogen or helium or a mixture of the two.

There exist in the solar system itself possible analogies to the mechanism by which emission luminosity may be excited in nebulosity by radiations emanating from a continuous source. Chief among these are the terrestrial aurorae, the solar corona, and the comets. In each of these cases, radiations of some sort emanating from the sun excite matter in a gaseous state to luminosity of a discontinuous nature. The prevailing opinion concerning the aurora is that the phenomenon is due to the ionization of atmospheric gases by high-speed charged particles.

H. N. Russell[1] has made suggestions of a similar nature to account for the spectrum of emission nebulae. Such a mechanism would account for the inverse-square law quite as well as a mechanism based on ethereal radiations, and would probably find no serious obstacle in the excessive luminosity of the planetaries. Furthermore, this explanation was also formulated from independent considerations, namely, the relations which Russell found to exist between relative sizes of monochromatic images of individual planetaries and ionization potentials of the radiations forming the images.

[1] *Observatory*, **44**, 72, 1921. Russell states that the luminosity of gaseous nebulae is probably due to ionization of the gases by radiations from associated stars, and that the radiations may be either ethereal or corpuscular.

Relations of a similar nature are suspected in the monochromatic images of diffuse nebulae with emission spectra. In these cases, however, the quantitative agreement between nebular and stellar luminosity, both for purely emission nebulae and for those with mixed spectra, appears to favor the notion of ethereal radiations. This agreement may of course be due to the nature of the relations between intensities of corpuscular radiations and spectral types or temperatures of the stars.

MOUNT WILSON OBSERVATORY
August 1922

NOTE ADDED DECEMBER 21, 1922

The stars within the great loop of nebulosity around the belt and sword of Orion sum up to an integrated apparent photographic magnitude of about 0.0. The loop extends to about 5°.7 or 342′ from the estimated center of gravity of the stars considered, and registers very nearly to this limit in exposures of about three hours. Therefore $m+5 \log a_1 = 11.5$, a value more representative than that derived from considering ζ Orionis alone.

New material on about 40 per cent of the objects in Curtis' list of planetary nebulae confirms the suggested systematic corrections for m and e as being of the right order of magnitude.

ORBIT OF THE ECLIPSING BINARY TW ANDROMEDAE[1]

By MARTHA BETZ SHAPLEY

ABSTRACT

Uniform and darkened orbits of the eclipsing binary TW Andromedae.—Orbits were calculated from 86 photographic observations made at Mount Wilson from 1914 to 1916. The adopted elements of light variation are

$$\text{Min.} = \text{J. D. } 2420051.618 + 4^{d}.122745 \text{ E}$$

The principal eclipse is total and there is some evidence of variation of the large, faint component. The *color index* at maximum is +0.57 mag., at minimum +1.36 mag. When reduced to the photographic scale by proper allowance for color, 74 photovisual observations, as well as the photographic measures, are well represented by the curves shown in Figs. 1 and 2, corresponding to the orbits whose elements are given in Table VI.

Photographic and photovisual observations of the eclipsing variable star TW Andromedae were made by Mr. Shapley with the 60-inch reflector at Mount Wilson during the years 1914 to 1916.

TABLE I

COMPARISON STARS FOR TW ANDROMEDAE

Star	Photographic Magnitude	Photovisual Magnitude	Color Index
B.D.+31°5023	10.70	9.99	$+0^{m}.71$
$23^{h}57^{m}$, +32°13′ (1900)	11.69	10.93	+0.76
B.D.+32°4755	9.27	8.81	+0.46

Miss Davis assisted in measuring and reducing the 160 multiple-exposure photographs, which contain a total of 540 images of the variable.

The magnitudes of the comparison stars, Table I, were standardized by means of the North Polar Standards.[2] Tables II and III contain the photographic and photovisual observations. The apertures employed range from nine inches to sixty inches; the exposure times from thirty seconds to two minutes.

[1] *Contributions from the Mount Wilson Observatory*, No. 254.

[2] Seares, *Mt. Wilson Contr.*, No. 97; *Astrophysical Journal*, **41**, 206, 1915.

TABLE II

PHOTOGRAPHIC OBSERVATIONS OF TW ANDROMEDAE

J.D. and Gr. H.M.T.	No. Exp.	Phase	Mag.	J.D. and Gr. H.M.T.	No. Exp.	Phase	Mag.
2420393.646....	5	−0^{d}160	10.26	2420455.651....	4	+0^{d}005	12.22
393.653....	5	−0.153	10.34	455.662....	4	+0.016	12.24
393.660....	5	−0.146	10.51	455.671....	4	+0.025	12.33
393.667....	5	−0.139	10.59	456.611....	5	+0.965	9.62
393.678....	5	−0.128	10.81	456.618....	4	+0.972	9.58
396.703....	4	+2.897	9.73	456.623....	4	+0.977	9.48
397.756....	5	−0.173	10.32	456.629....	4	+0.983	9.46
397.770....	5	−0.159	10.26	458.617....	4	+2.970	9.84
397.786....	5	−0.143	10.53	458.626....	4	+2.979	9.76
397.797....	5	−0.132	10.51	459.578....	4	−0.192	9.82
397.812....	5	−0.117	10.70	459.584....	4	−0.186	10.13
397.837....	5	−0.092	11.26	459.590....	4	−0.180	10.02
397.863....	5	−0.066	11.66	459.597....	4	−0.173	10.18
397.876....	5	−0.053	11.88	459.602....	4	−0.168	10.23
397.889....	5	−0.040	12.09	459.607....	4	−0.163	10.17
397.903....	5	−0.026	11.98	459.615....	4	−0.155	10.31
397.916....	5	−0.013	12.00	459.624....	4	−0.146	10.24
397.929....	5	0.000	12.06	459.637....	4	−0.133	10.51
397.944....	5	+0.015	12.29	459.647....	4	−0.123	10.67
397.958....	5	+0.029	12.16	459.656....	4	−0.114	10.68
397.973....	5	+0.044	12.12	459.667....	4	−0.103	10.88
397.986....	5	+0.057	11.77	459.679....	4	−0.091	11.04
398.003....	5	+0.074	11.53	459.689....	4	−0.081	11.35
398.017....	5	+0.088	11.32	459.698....	4	−0.072	11.52
398.031....	5	+0.102	11.03	459.727....	4	−0.043	11.98
398.035....	5	+0.107	10.88	459.736....	4	−0.034	12.04
422.707....	3	+0.042	12.29	459.745....	4	−0.025	12.12
422.734....	3	+0.069	11.56	517.669....	4	+0.181	10.01
422.746....	4	+0.081	11.38	517.676....	4	+0.188	9.91
422.757....	4	+0.092	11.21	517.687....	4	+0.199	9.71
422.777....	4	+0.112	10.81	517.694....	4	+0.206	9.65
422.787....	4	+0.122	10.53	517.704....	4	+0.216	9.64
422.797....	4	+0.132	10.36	517.712....	4	+0.224	9.73
422.811....	4	+0.146	10.52	2421050.911....	3	+1.589	9.86
425.622....	4	+2.957	10.07	050.924....	3	+1.602	9.88
425.631....	4	+2.966	10.00	051.864....	3	+2.542	9.68
425.675....	4	+3.010	9.68	051.889....	3	+2.567	9.84
425.683....	4	+3.018	9.55	052.863....	3	+3.541	9.78
425.692....	4	+3.027	9.56	052.891....	3	+3.569	9.78
425.699....	4	+3.034	9.59	053.867....	3	+0.422	9.78
425.707....	4	+3.042	9.57	053.876....	3	+0.431	9.80
425.716....	4	+3.051	9.56				
455.623....	4	−0.023	12.25				
455.632....	4	−0.014	12.16				
455.642....	4	−0.004	11.99				

TABLE III

PHOTOVISUAL OBSERVATIONS OF TW ANDROMEDAE

J.D. and Gr.H.M.T.	No. Exp.	Phase	Mag.	J.D. and Gr. H.M.T.	No. Exp.	Phase	Mag.
2420396.687....	3	$+2^{\mathrm{d}}871$	9.06	2420458.622....	3	$+2^{\mathrm{d}}975$	9.23
397.749....	3	−0.180	9.71	458.631....	3	+2.984	9.31
397.762....	3	−0.167	9.67	459.582....	3	−0.188	9.34
397.776....	3	−0.153	9.64	459.587....	3	−0.183	9.36
397.790....	3	−0.139	9.78	459.594....	3	−0.176	9.45
397.803....	3.	−0.126	9.76	459.599....	3	−0.171	9.54
397.819....	3	−0.110	10.02	459.605....	3	−0.165	9.44
397.858....	3	−0.071	10.45	459.610....	3	−0.160	9.54
397.870....	3	−0.059	10.40	459.619....	3	−0.151	9.51
397.883....	3	−0.046	10.57	459.628....	3	−0.142	9.49
397.897....	3	−0.032	10.85	459.642....	3	−0.128	9.63
397.910....	3	−0.019	10.78	459.651....	3	−0.119	9.87
397.923....	3	−0.006	11.19*	459.660....	3	−0.110	10.03
397.936....	3	+0.007	10.77	459.674....	3	−0.096	10.23
397.952....	3	+0.023	10.87	459.685....	3	−0.085	10.25
397.966....	3	+0.037	10.68	459.694....	3	−0.076	10.42
397.979....	3	+0.050	10.64	459.702....	3	−0.068	10.63
397.995....	3	+0.066	10.60	459.731....	3	−0.039	10.90
398.009....	3	+0.080	10.40	459.740....	3	−0.030	10.77
398.024....	3	+0.095	10.15	459.749....	3	−0.021	10.88
422.728....	3	+0.063	10.82	517.673....	3	+0.185	9.47
422.740....	3	+0.075	10.64	517.681....	3	+0.193	9.23
422.752....	3	+0.087	10.35	517.691....	3	+0.203	9.31
422.769....	3	+0.104	10.08	517.698....	3	+0.210	9.13
422.783....	3	+0.118	9.81	517.708....	3	+0.220	9.19
422.792....	3	+0.127	9.71	517.715....	3	+0.227	9.38
422.801....	3	+0.136	9.72	2421050.906....	2	+1.584	9.08
425.626....	3	+2.961	8.85:	050.919....	2	+1.597	9.18
425.635....	3	+2.970	9.13	051.857....	2	+2.535	9.27
425.679....	3	+3.014	9.07	051.885....	2	+2.563	9.13
425.686....	3	+3.021	9.08	052.858....	2	+3.536	9.13
425.696....	3	+3.031	9.02	052.886....	2	+3.564	9.28
425.703....	3	+3.038	9.20	053.872....	2	+0.427	9.16
425.712....	3	+3.047	9.08	053.881....	2	+0.436	9.23
425.721....	3	+3.056	9.16				
425.799....	4	+3.134	9.16				
456.615....	3	+0.969	9.02				
456.621....	3	+0.975	9.10				
456.626....	3	+0.980	9.11				
456.632....	3	+0.986	9.02				

* Rejected observation, abnormality unexplained.

The period used for the reduction is that derived by van der Bilt.[1] To satisfy the Mount Wilson observations, a correction of $-0^d.004$ has been made to the initial epoch, giving the adopted elements:

$$\text{Min.} = \text{J.D. } 2420051.618 + 4^d.122745\ E.$$

The individual photographic observations are plotted in Figure 1 with the curve computed from orbital elements, based on the assumption of disks of uniform brightness. The scattering of the observations at the bottom of the minimum is believed to be due, in part at least, to actual fluctuations in the light of the presumably very red faint component, which alone is visible at the time of principal eclipse.

TABLE IV

PHOTOGRAPHIC NORMAL POINTS FOR TW ANDROMEDAE

Number	No. Obs.	Mean Phase	Mean Mag.	O–C, Darkened
1	13	$0^d.018$	12.14	$0^m.00$
2	5	.044	12.07	+0.05
3	5	.068	11.61	0.00
4	5	.087	11.27	+0.01
5	5	.103	10.96	−0.01
6	5	.121	10.68	−0.01
7	5	.136	10.50	0.00
8	5	.149	10.38	+0.02
9	5	.165	10.25	+0.05
10	5	.182	10.05	−0.01
11	5	0.207	9.71	−0.19
12	23	....	9.70	0.00

Tables IV and V contain the mean magnitudes, photographic and photovisual, after reflecting all observations on to one branch of the curve. The first normal group of each table contains the observations during the phase of totality; the last includes all observations outside the minimum.

The orbits are based on the photographic light curve.[2] Uniform and darkened elements were computed, and, together with other data concerning the system, are arranged in Table VI. The mean densities are not corrected for probable unequal distribution of mass.

The photographic normals are plotted as dots in Figure 2, where the co-ordinates are, respectively, light intensity, corresponding to

[1] *Astronomische Nachrichten*, **196**, 396 (No. 4703), 1913.

[2] A provisional orbit has been published by Stewart in *Astrophysical Journal*, **42**, 315, 1915.

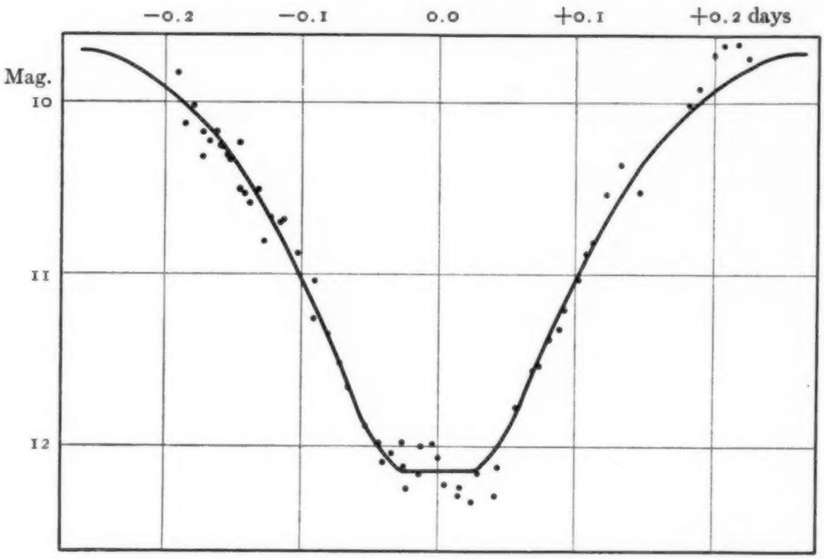


FIG. 1.—Photographic observations and computed uniform light-curve for TW Andromedae.

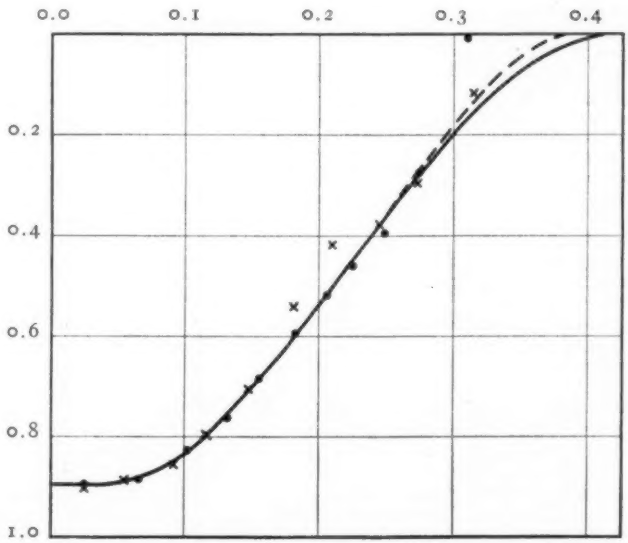


FIG. 2.—Computed intensity-curves for TW Andromedae. Ordinates are losses of light-intensity, $1-l$; abscissae are sines of phase angles.

TABLE V

PHOTOVISUAL NORMAL POINTS FOR TW ANDROMEDAE

Number	No. Obs.	Mean Phase	Mean Mag.	O-C, Darkened
1	4	$0^{d}018$	10.82	$+0^{m}04$
2	5	.037	10.75	−0.01
3	5	.061	10.62	+0.03
4	5	.077	10.43	0.00
5	5	.098	10.17	−0.02
6	5	.120	9.83	−0.11
7	5	.139	9.63	−0.13
8	5	.163	9.57	0.00
9	5	.182	9.47	+0.02
10	5	0.211	9.25	−0.04
11	24	....	9.13	0.00

TABLE VI

ORBITAL ELEMENTS AND OTHER DATA FOR TW ANDROMEDAE

Right Ascension	$23^{h}58^{m}10^{s}$ } 1900	
Declination	$+32°17'.3$ } 1900	
Period	$4^{d}122745$	
Nature of eclipse	Total	
	Pg.	Pv.
Magnitude at maximum	9.70	9.13
Magnitude at minimum	12.14	10.78
Range at primary minimum	$2^{m}44$	$1^{m}65$
Light of brighter star	0.894	0.781
Light of fainter star	0.106	0.219
Color index at maximum	$+0^{m}57$	
Color index at minimum	+1.36	
	Uniform	Darkened
Inclination of orbit	81°40′	86°2′
Least distance of centers	0.145	0.069
Semi-duration of primary eclipse	$0^{d}256$	$0^{d}285$
Semi-duration of totality	$0^{d}026$	$0^{d}023$
Computed range at secondary minimum	$0^{m}02$	$0^{m}06$
Radius of brighter star	0.127	0.174
Radius of fainter star	0.275	0.252
Ratio of radii	0.46	0.69
Most probable density of brighter star	0.193	0.075
Most probable density of fainter star	0.019	0.025
Radius of brighter star in terms of sun's radius	1.73	2.36
Radius of fainter star in terms of sun's radius	3.74	3.42
Distance of centers in terms of sun's radius	13.6	13.6
Ratio of surface brightness in photographic light	40.	17.7
Photovisual absolute magnitude of brighter star	1.4	0.7
Photovisual absolute magnitude of fainter star	2.9	2.2
Hypothetical parallax	$0''003$	$0''002$

the mean magnitude, and the sine of the orbital longitude, corresponding to the mean phase. The full-line curve is computed from the darkened elements; the broken curve, representing the uniform orbit, coincides with it throughout more than half the eclipse.

The photovisual normals have been reduced to the photographic light-curve by applying as a factor the ratio of photographic to photovisual range at primary minimum; they are represented by crosses in Figure 2. It is clear from the plot that the orbit derived from photographic observations satisfactorily represents both the photographic and photovisual data, when allowance is thus made for the difference in the color of the two components.

Cambridge, Massachusetts
March 1922

ORBITS OF THE SPECTROSCOPIC BINARIES LALANDE 13792 AND AOe 12584[1]

BY R. F. SANFORD

ABSTRACT

Orbits of the spectroscopic binaries Lalande 13792 and AOe 12584.—Both binaries are dwarfs, with absolute magnitudes of about +5. The elements of each, determined from more than twenty spectrograms, are respectively: P, 32.8092 and 5.4145 days; e, 0.080 and 0.000; K, 27.5 and 64.9 km/sec; γ, +19.7 and −97.4 km/sec. In the case of AOe 12584, the lines of the secondary show faintly and give for K, +74.0 km/sec, and for the functions $m \sin^3 i$ and $m_1 \sin^3 i$ the values 0.803 and 0.704⊙. The large space velocity of AOe 12584, like that of Lalande 29330, is directed away from the quadrant which the apices of other stars of large space motion in general avoid.

Differences between dwarf and giant binaries.—The six spectroscopic binaries of the dwarf division whose orbits have been determined by the writer display smaller eccentricities for a given period group than do giant binaries, as has been shown by R. E. Wilson from data relating to both visual and spectroscopic binaries. Furthermore, they have a mean period much shorter than that of the majority of giants of the same spectral division.

The orbits of nine spectroscopic binaries determined by the writer at the Mount Wilson Observatory have appeared in two previous papers.[2] In the introduction to the first of these may be found explanations of the tables of observations, methods of obtaining and correcting the preliminary elements, and the formation of the velocity curves. In general these statements apply to the two

TABLE I

Name	Mag.	α(1900)	δ(1900)	Spectral Class	Vis. Abs. Mag.	μ	π Sp.	No. Rev.
Lal. 13792=H.D. 54371...	7.0	$7^h\ 3^m5$	+25°54′	G5	4.8	0″225	0″036	26
AOe 12584=H.D. 107760..	8.2	12 17.9	+73 48	G6	5.4	0.49	0.028	340

H.D.=Henry Draper Memorial Catalog.

spectroscopic binaries whose orbits are presented in the present contribution. All of the data in Table I are taken from the list of spectroscopic parallaxes of 1646[3] stars. The last column shows

[1] *Contributions from the Mount Wilson Observatory*, No. 251.

[2] *Ibid.*, Nos. 201, 221; *Astrophysical Journal*, **53**, 201, 1921; **55**, 30, 1922.

[3] *Mt. Wilson Contr.*, No. 199; *Astrophysical Journal*, **53**, 13, 1921.

the number of revolutions of each star in its orbit between the first and last observation of velocity recorded in the tables of observations.

LALANDE 13792 (B.D.+25°1594)

This star was placed on the spectroscopic observing list to increase the data for the curves used for the determination of absolute magnitudes, since the parallax, 0".031, derived at the Allegheny Observatory, together with the apparent magnitude, showed it to be a star of low absolute magnitude. The annual

TABLE II

OBSERVATIONS OF LALANDE 13792

Plate No.	Date	G.M.T.	Phase	Velocity	O−C
				km/sec	km/sec
γ 8947	1920, Jan. 6	22h02m	13d588	+ 1.4	−3.0
9070	Mar. 11	18 32	12.825	+ 5.2	+3.5
9600	Sept. 29	0 07	17.202	+19.2	−0.6
9715	Nov. 3	0 35	19.411	+32.6	+3.0
9733	Nov. 21	0 55	4.616	+ 1.8	+3.0
9836	Dec. 27	23 50	8.762	− 5.8	+1.5
9941	1921, Feb. 23	20 14	0.994	+13.4	−4.3
10007	Mar. 17	17 50	22.894	+33.3	−9.2
C 955	Mar. 20	17 10	25.866	+51.2	+3.8
γ 10102*	Apr. 19	16 23	23.025	+47.7	+4.9
10111*	Apr. 21	17 42	25.080	+37.7	−9.2
10681	Dec. 8	1 00	25.719	+53.6	+6.2
10754*	1922, Jan. 12	22 26	28.803	+47.4	+3.9
C 1527	Jan. 13	23 43	29.956	+39.5	−0.2
γ 10766	Jan. 15	17 13	31.586	+31.9	+0.9
10825	Feb. 14	18 32	28.831	+38.1	−5.3
C 1567	Feb. 15	19 00	29.851	+41.6	+1.9
γ 10846	Feb. 18	15 24	32.701	+26.0	+1.5
10861	Mar. 6	18 26	16.018	+12.0	−2.5
C 1596	Mar. 8	19 25	18.060	+27.6	+4.0
γ 10918	Mar. 17	17 35	26.982	+47.2	+0.1
11004	May 4	16 14	9.308	−10.6	−3.8

* Given weight ½ in least-squares solution.

proper motion, 0".225, is in keeping with this conclusion. The parallax derived by the spectroscopic method is 0".036, in good agreement with the trigonometric value. The variability in radial velocity was strongly suspected from the measures of the third plate, and definitely confirmed by that of the fourth. Twenty-two spectrograms have been secured. (Table II.)

The estimated spectrum is G5, with lines of satisfactory quality. There is no certain evidence of the spectrum of the secondary star. It was found possible to assemble all the velocities upon a satisfactory preliminary curve with 32.8092 days as the period. The twenty-six orbital revolutions which separate the first and last observations make an interval so long that only very small changes in the period could be made without seriously disturbing the velocity-curve. Hence this period has been considered final.

Preliminary elements derived by Russell's method and by the graphical method of Lehmann-Filhés were in substantial agreement, the latter giving

PRELIMINARY ELEMENTS

P	32.8092 days
e	0.052
ω	99°.7
K	+28.3 km/sec
T	J.D. 2423073.7
γ	+21.2 km/sec

An ephemeris computed with these elements gave the residuals for the velocities. Three plates as indicated in Table II were assigned weight one-half on account of their poor quality. All others were given weight unity.

The final elements result from the correction of the preliminary value by two least-squares solutions, the second being undertaken

FINAL ELEMENTS AND PROBABLE ERRORS

P	32.8092 days
e	0.080±0.039
ω	82°.1±26°.1
K	27.5±1.0 km/sec
T	J.D. 2423071.941±2^d.334
γ	+19.7 km/sec
$a \sin i$	12,383,000 km
$\frac{m_1^3 \sin^3 i}{(m+m_1)^2}$	0.0705 ⊙

because of rather large differences between the residuals derived from the elements as first corrected and those derived by substitution of the unknowns in the conditional equations. Although the

quantity Σpv^2 for the final elements, which is seventy-two per cent of its value for the preliminary elements, scarcely differs from that for the elements as first corrected, the ephemeris and conditional equations give residuals that are in satisfactory agreement; moreover, the probable errors for ω and T have been materially reduced, although those for K and e remain as before. The probable error for a velocity of unit weight is ± 3.0 km/sec.

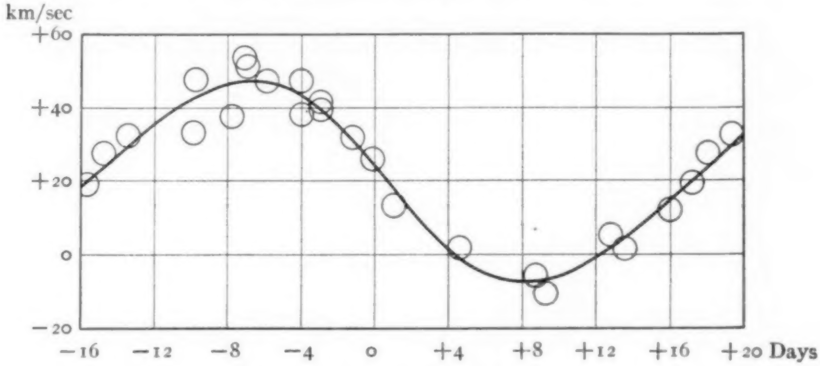


FIG. 1.—Velocity-curve of Lalande 13792

In the radial-velocity diagram (Fig. 1) the velocity of the system has not been represented in the usual way, since its value, +19.7 km/sec, is so nearly that of the line whose ordinate is +20 km/sec that, with the scale used, no distinction could be made between the two.

AOe 12584 (B.D.+74°493)

This star was placed on the Mount Wilson observing list for the determination of absolute magnitudes by the spectroscopic method because of its large annual proper motion (0".49) and its apparent magnitude (8.2), which suggested low luminosity. The observations gave an absolute magnitude of +5.4, spectral class G6, and the second spectrogram revealed the variation in its radial velocity. Twenty-four plates (Table III) have been secured with the 60-inch reflector, using a one-prism spectrograph provided with a camera of 18-inches focus, except that for the second, third, and fourth plates a camera of 7-inches focus was used. These have been assigned half weight. The northerly declination of this object

puts it beyond the reach of the Hooker telescope, the use of which would have been advantageous, for with the 60-inch reflector, even under fair conditions, it has been necessary to use exposures of about three hours. Under poor conditions, which are likely to occur at the season when the star is best placed for observation, longer exposures have been necessary and some plates are rather

TABLE III

OBSERVATIONS OF AOE 12584

Plate No.	Date	G.M.T.	Phase	Velocity Prim.	Velocity Second	O−C Prim.	Wt. Prim.
				km/sec	km/sec	km/sec	
γ 5704......	1917, Apr. 4	$20^h 20^m$	$2^d.042$	−152.9	....	− 9.0	0.5
6929*.....	1918, May 24	17 25	1.001	− 70.3	....	+ 1.2	0.5
8170*.....	1919, May 13	19 25	3.139	−170.6	....	−16.2	0.5
8292*.....	June 13	17 23	1.567	−106.1	....	+ 7.2	0.5
9056......	1920, Mar 8	20 30	5.385	− 33.1	−163.2	− 0.5	1.0
10000......	1921, Mar. 16	21 02	4.803	− 44.1	−149.1	+ 4.0	1.0
10008......	Mar. 17	20 23	0.362	− 37.7	−168.0	+ 0.4	1.0
10100......	Apr. 18	19 49	5.267	− 38.6	−156.6	− 5.2	0.5
10113......	Apr. 21	21 00	2.901	−151.8	− 28.0	+ 8.9	1.0
10125......	Apr. 25	20 18	1.457	−102.1	....	+ 3.1	1.0
10148......	May 14	18 45	4.148	− 96.4	....	− 5.6	1.0
10152......	May 15	19 22	5.174	− 37.2	−167.8	− 2.3	1.0
10200......	June 14	18 10	2.637	−169.4	....	− 7.3	0.5
10202......	June 15	18 00	3.630	−123.8	− 33.6	+ 4.7	0.5
10206......	June 16	17 58	4.629	− 59.1	....	− 1.5	0.5
10210......	June 18	17 39	1.200	− 87.6	....	− 1.7	1.0
10220......	June 20	17 36	3.199	−158.5	− 31.0	− 6.4	1.0
10251......	July 11	17 23	2.532	−158.0	− 33.6	+ 2.9	1.0
10255......	July 12	17 10	3.523	−119.3	....	+16.1	1.0
10262......	July 13	17 07	4.521	− 59.1	−135.4	+ 5.2	1.0
10277......	July 16	17 06	2.105	−148.8	− 27.7	− 1.7	1.0
10862......	1922, Mar. 6	21 45	2.474	−158.6	− 27.2	+ 1.3	0.5
10963......	Apr. 7	19 26	1.891	−137.4	....	− 2.1	0.5
11000......	Apr. 18	18 29	2.021	−143.7	− 20.2	− 0.8	1.0

* Taken with 7-inch camera.

underexposed. Weights have been assigned accordingly. The spectrum shows a satisfactory set of lines for measurement when the velocity is near that of center of mass. But the secondary spectrum is bright enough to render the spectral lines of the primary less distinct by the superposition of the continuous spectrum when the radial velocities of the two components differ. Measures of the radial velocity of the secondary have been made on several plates, as Table III shows, but are not considered suitable to the

determination of any element except the amplitude and the relative masses of the two components, to be referred to later.

A set of preliminary elements, derived after twenty spectrograms had been secured, has already been published.[1] One branch of the velocity-curve was later strengthened by four additional plates, after which a second set of elements was derived.

PRELIMINARY ELEMENTS

	(1)	(2)
P	$5^{\mathrm{d}}414$	$5^{\mathrm{d}}41454$
e	0.000	0.025
ω		204°
K	64 km/sec	67 km/sec
K_1	70 km/sec	
T	J.D. 2422798.875	2422856.162
γ	−97.5 km/sec	−97.8 km/sec

The differences are slight, when it is considered that for circular elements the epoch is arbitrary and that K_1 has been left for determination in a more accurate way.

The second set indicated a slight eccentricity, the reality of which it seemed more satisfactory to leave to the outcome of a least-squares solution. Since 340 revolutions separate the first and last observations, the period has been taken as definitive. If K_1 is derived independently, and ω_1 for the secondary is taken as $\omega+180°$, there remain only five elements to be corrected by the method of least-squares.

As often happens when the eccentricity is small, two of the resulting normal equations were practically identical. Hence only four, instead of five unknowns, could be determined satisfactorily. In this case, instead of corrections to ω and T, a correction to u, the argument of latitude, results. As a matter of fact, three sets of corrections have been derived, (*a*) one which corrects all five elements, (*b*) another set which corrects three elements and the argument of latitude, u, and (*c*), a set like the second, except that it is based upon the twenty-two spectrograms that remain after the rejection of two (γ 8170 and γ 10255) with large residuals. The

[1] *Publications American Astronomical Society*, **4**, 283, 1921.

value of Σpv^2 for the elements thus corrected becomes less and less, both for all the observations and for the twenty-two considered in the last set. In other words, the rejection of the two observations showing large residuals leads to corrections which do not materially affect the size of these residuals and does improve the representation of the remaining observations.

A comparison of the residuals derived from an ephemeris based upon set (*c*) with those derived from substitution into the conditional equations showed that the elements might be further improved. Since these corrections had reduced the eccentricity to 0.005, this element was assumed to be zero and another least-squares solution for corrections to γ, K, and T was made, T now being referred to the time of the smallest negative velocity of the primary. After these corrections had been applied, ephemeris and conditional equations gave practically identical values for the residuals. Hence these elements are considered definitive.

K_1 has been determined from the relation $K_1 = K\left(\frac{V_1-\gamma}{V-\gamma}\right)$ in which V is the computed velocity for the primary, V_1 is the observed velocity of the secondary, and γ and K are elements of the primary. The weights assigned depend on the number of lines measured in determining V_1.

FINAL ELEMENTS AND PROBABLE ERRORS

P	$5^d.41454$
K	64.9 ± 1.3 km/sec
K_1	74.0 km/sec
T	J.D. $2422853.120 \pm 0^d.020$
γ	-97.4 km/sec
$a \sin i$	4,833,000 km
$a_1 \sin i$	5,510,000 km
$m \sin^3 i$	$0.803 \odot$
$m_1 \sin^3 i$	$0.704 \odot$

Reference to set (1) of the preliminary elements will show how closely they agree with the final elements, and that the choice of set (2) was not a happy one, although for this case the eccentricity was not arbitrarily assumed to be zero.

If all twenty-four plates are considered, the probable error of a single determination of radial velocity of unit weight is ±4.0 km/sec, and ±2.9 km/sec if based upon the twenty-two plates finally used. The probable errors for the elements are based on the larger of these two values, which is also used as radius for the circles representing the individual velocities in the diagram in Figure 2. The barred circles refer to velocities derived from the three spectrograms taken with low dispersion to which reference

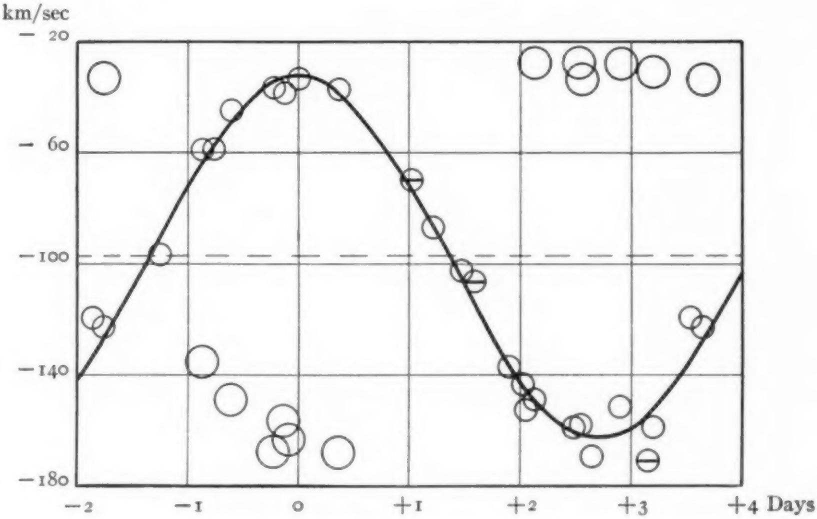


FIG. 2.—Velocity-curve of AOe 12584

has already been made. The larger circles represent the velocities for the secondary star as far as they were measured. Their evident inaccuracy did not seem to warrant the inclusion of a velocity-curve for them.

Including the two binaries discussed in this paper, six of a total of eleven whose orbits have been derived by the writer at Mount Wilson have absolute magnitudes that place them in the dwarf division. Since binary stars of low luminosity are relatively few in number, it is of interest to collect certain elements for these six stars, as has been done in Table IV.

Attention has already been called to the fact that the dwarf binary Lalande 29330[1] has a period and an eccentricity both of which are small for a star of its spectral class when compared with mean values based upon hitherto existing material, relating, for the

TABLE IV

Name	App. Mag.	Sp. Class	Abs. Mag.	P	e	K	γ	$a \sin i$	$\frac{m_2^3 \sin^3 i}{(m+m_1)^2}$
						km/sec	km/sec	km	⊙
OΣ 82.....	7.0	F9	4.1	$4^d.0000$	0.060	36.1	+37.4	1.98×10^6	0.0193
Lal. 29330..	8.5	K0	6.0	4.2850	0.089	38.1	−60.6	2.24×10^6	0.0244
Lal. 49867..	7.3	K2	6.0	6.7217	0.059	38.5	−19.8	3.55×10^6	0.0396*
AOe 12584..	8.2	G6	5.4	5.4145	0.000	64.9	−97.4	4.83×10^6	....
75 Cancri...	6.0	G2	4.1	19.4589	0.206	20.2	+12.3	5.26×10^6	0.0157
Lal. 13792..	7.0	G5	4.8	32.8092	0.080	27.5	+19.7	12.38×10^6	0.0705

* Given erroneously as 0.0125 ⊙ in *Mt. Wilson Contr.*, No. 201.

most part, to giant stars. Forming mean values of spectral class, period, eccentricity and absolute magnitude for the six binaries, we find the data of the first line of Table V. This is followed by the similar data for class G stars taken from a table by Aitken.[2]

TABLE V

Spectral Class	Period	Eccentricity	Abs. Mag.
G4..............	12 days	0.082	+5.1
G...............	267 days	0.129	Mostly giants

When it is considered that the first line shows mean values for stars whose spectral classes range from F9 to K2, and that in the table from which the second line is taken the value of e is larger for the spectral divisions which precede and follow G, it appears that in Table V we again have evidence that dwarf binaries have shorter periods and smaller eccentricities than do giant stars of approximately the same spectral class.

Dr. R. E. Wilson has also discussed the period-eccentricity relation[3] for binary stars, and as far as his data go, finds that for the

[1] *Mt. Wilson Contr.*, No. 201; *Astrophysical Journal*, **53**, 212, 1921.

[2] *The Binary Stars*, p. 199, 1918.

[3] *Astronomical Journal*, **33**, 147, 1921.

same average period dwarf stars have an average eccentricity smaller than that of the giants. In order to display this feature, the material in Table I of his paper corresponding to that in Table IV above is reproduced in Table VI. The first column gives his number of the period group, the range in each of which is given in the second column. Then follow pairs of columns which show the mean eccentricity and the number of stars for giants and for dwarfs, and, finally a pair which give the same data derived from the six binaries in Table IV.

TABLE VI

Group	Period	Giants		Dwarfs		Dwarfs (Sanford)	
		e	No.	e	No.	e	No.
2	4– 8 days	0.114	19	0.020	3	0.052	4
3	8– 25 days	0.250	22	0.132	4	0.206	1
4	25–100 days	0.468	15	0.010	1	0.080	1

It is readily seen that the eccentricities for the first column of dwarfs are systematically smaller than those for giants, which is borne out rather well by the spectroscopic binaries whose eccentricities are displayed in the next to the last column.

Both Aitken[1] and Wilson[2] have called attention to an apparently abrupt change in the mean eccentricity for the stars with periods between 25–100 days.

Although the data for such a conclusion are meager indeed, the same phenomenon is shown in the last column of eccentricities, but it is by no means certain from either the fifth or seventh columns that there is a minimum in the eccentricity-period relation for group 4 or a maximum for group 3. Two more dwarf binaries for which I have nearly enough material for orbits will add one star each to groups 2 and 3 and will have eccentricities consistent with the values given in this seventh column. Orbital elements of dwarf spectroscopic binaries falling in groups 3 and 4 are therefore especially to be desired in order to settle (*a*) the reality of this departure from a smooth curve in the period-eccentricity relation, and (*b*) whether the departure means a maximum for group 3 or a minimum for group 4.

[1] *The Binary Stars*, p. 197, 1921.

[2] *Astronomical Journal*, **33**, 147, 1921.

It is of interest to note that two of the binaries of Table IV, Lalande 29330 and AOe 12584, have large velocities of the center of mass (γ). Their space motions place them in that group of stars of large space-velocity whose motions have been discussed by Adams and Joy[1] and later more completely by Strömberg,[2] and whose apices are directly away from the quadrant of avoidance shown to exist by these investigators.

MOUNT WILSON OBSERVATORY
September 1922

[1] *Mt. Wilson Contr.*, No. 163; *Astrophysical Journal*, **49**, 179, 1919.

[2] *Proceedings of the National Academy of Sciences*, **8**, 141, 1922.

STELLAR SPECTRA OF CLASS S[1]

I. GENERAL DESCRIPTION

By PAUL W. MERRILL

ABSTRACT

Class S stars.—A number of red stars have spectra similar to that of R Geminorum, which differs from the recognized types of the Harvard classification. It is proposed to group them into an additional class with the symbol S. *A provisional list* of 22 S stars is given, including the following *long-period variables: X and R Androm., R and T Camelop., V Cancri, R Can. Min., U and S Cassiop., R Cygni, R and T Gemin., RW Librae, R Lyncis, R Orionis, T Sagittarii, and S Urs. Maj. Characteristic spectral features* are shown by typical spectrograms and by a table of forty-three lines from λ 4500 to λ 4860, particularly a complicated structure between λ 4630 and λ 4660. The absorption lines λ 4554 (Ba) and λ 4607 (Sr) are unusually strong, and there is an absorption band in the red extending from λ 6470 to longer wave-lengths. A remarkable feature is the presence of enhanced lines of iron, $\lambda\lambda$ 4584, 4924, and 5018, as emission lines. As in the case of class M stars, the variables alone show strong bright hydrogen lines (type Se); $H\beta$ is more intense than $H\gamma$, and the lines are narrow. A list of the plates obtained at Mount Wilson, together with the dates, phases, and notes as to the individual spectra, are given. *The relationship to other classes* is discussed. While the S stars appear to be more closely related to giant Ma stars than to any other type, they probably do not belong in either the G-R-N or the G-K-M branch, but form a third branch. The recognition of these stars as a distinct group should aid in the general study of long-period variables.

In Secchi's classification of stellar spectra, made nearly sixty years ago, there are two great groups of orange and red stars, namely, types III and IV. Both types are marked by characteristic absorption bands which face in opposite directions in the two types. Those of type III are now known to be due to titanium oxide, and those of type IV to carbon. In the more detailed system of classification developed at the Harvard College Observatory, these groups, called classes M and N respectively, have been subdivided, but stars with intermediate spectra have not been found. No object is known to show both carbon and titanium bands; these spectral features seem to be mutually exclusive. Accordingly there is no place for the carbon stars in the spectral progression which runs without a break through the sequence of classes B, A, F, G, K, and M, since the latter end of this chain is characterized by increasing strength of the titanium bands. The alternative arrangement for class N is a side branch paralleling the main

[1] *Contributions from the Mount Wilson Observatory*, No. 252.

sequence in the K-M region. The discovery of the class R stars, which in color and spectroscopic details are intermediate between classes G and N, strongly supports this view.[1]

Among the long-period variables there are a number which are not clearly of either class M or class N, and which have been variously classified by the Harvard observers. The following statement in regard to these stars is found in the introduction to the *Henry Draper Catalogue:*

> Several spectra which have hitherto been called Md1 or Md2, in which Hβ is the strongest bright line, are found to be peculiar and are designated Pec. in Table I. The variable stars R Andromedae, U Cassiopeiae, S Cassiopeiae, R Lyncis, R Canis Minoris, T Geminorum, and R Cygni may be given as examples. These spectra do not show the titanium bands having bright edges at 4762, 4954, and 5168 as in all divisions of Class M, but more nearly resemble the spectrum of π^1 Gruis, which may be placed in a subdivision of Class R, assuming some peculiarities.

Few observations of any of these stars have been made with slit spectrographs until recently. Spectrograms of R Cygni were secured by Wright[2] in December, 1911. They showed a complicated spectrum most of whose details could not be identified. Several stars of this type were included in the radial velocity observations of long-period variables[3] made by the writer at Ann Arbor from 1913 to 1915, but in only one case was the exposure sufficient to show the absorption spectrum; R Lyncis, photographed in February, 1915, was found to have a spectrum resembling that of R Cygni as observed by Wright.

Since resuming the investigation of long-period variables at Mount Wilson, I have photographed a number of objects which have spectra of the same type as R Cygni and R Lyncis. They have been recognized as forming a special group distinct from either class M, N, or R. The designation class "S" has been suggested for them by the action of the International Astronomical Union at the Rome meeting in May, 1922.

[1] The relationships between classes G, R, and N have been thoroughly discussed by Rufus, *Publications of the Observatory, University of Michigan*, **2**, 103, 1916.

[2] *Monthly Notices*, **72**, 548, 1912. The preceding article gives an account of visual observations by Espin.

[3] *Publications of the Observatory, University of Michigan*, **2**, 45, 1916.

It is the purpose of this article to furnish a description of spectra of class S together with some remarks on the relationship to other classes. The general conclusion may be anticipated here by stating that the S stars probably form a third branch of the spectral sequence in addition to the G-K-M and the G-R-N branches. (See page 473.)

In Table I is given a first list of stars of class S. The observations upon which the inclusion of each star depends may be inferred

TABLE I

PROVISIONAL LIST OF STARS OF CLASS S

Name	H.D.	R.A. 1900	Dec. 1900	Mag.	Spect.	Period
						days
X Androm.	1167	0^h10^m9	$+46°27'$	8.1–14.2	Se	346.5
R Androm.	1967	0 18.8	+38 1	5.6–14.0	Se	410.7
U Cassiop.	4350	0 40.8	+47 43	8.0–16.	Se	276.0
S Cassiop.	7769	1 12.3	+72 5	7.6–14.5	Se	609.5
T Camelop.	29147	4 30.3	+65 57	7.0–13.5	Se	370
R Orionis	31798	4 53.6	+ 7 59	8.7–13.5	Se?	378.5
	35155	5 17.6	− 8 45	7.0	S	....
R Lyncis	51610	6 53.0	+55 27	7.0–13.8	Se	379.2
R Gemin.	53791	7 1.3	+22 52	6.4–13.8	Se	370
R Can. Min.	54300	7 3.2	+10 11	7.2–10.0	Se	337.7
	58881	7 22.4	−11 31	9.0	S	....
T Gemin.	63334	7 43.3	+23 59	8.0–13.5	Se	288.1
	63733	7 45.3	−18 45	8.5	S	....
V Cancri	70276	8 16.0	+17 36	7.5–13.0	Se	272.1
S Urs. Maj.	110813	12 39.6	+61 38	7.3–12.5	Se	226.5
	121447	13 50.3	−17 45	8.1	S	....
R Camelop	127226	14 25.1	+84 17	7.9–13.7	Se	269.5
RW Librae	136734	15 17.2	−23 42	8.6–11.7	Se	490 (about)
	172804	18 37.1	+ 6 43	9.1	S	....
T Sagittarii	....	19 10.5	−17 9	7.2–12.0	Se	381.3
R Cygni	....	19 34.1	+49 58	6.6–13.9	Se	426
π^1 Gruis	....	22 16.6	−46 27	6.6	S	....

from Table II, which gives a record of the Mount Wilson spectrograms, and from the Harvard notes and the brief discussions of each star which follow. It is highly probable that future observations will disclose additional stars of class S, especially among the long-period variables. It is less likely, though not impossible, that one or two stars of Table I will be shown by improved observations to belong to some other spectral class.

Table II contains a record of most of the Mount Wilson spectrograms of stars of class S. They were secured with one-prism slit

TABLE II

JOURNAL OF OBSERVATIONS

Star	Plate	Date	Mag.	Phase	Intensities Hβ	Hγ	Hδ	Remarks
				days				
X Androm.	C 1454	1921, Nov. 13	9.5	−16	12	2	.....	
	C 1462	Nov. 14	9.5	−15	4	0.5	.....	
R Androm.	γ 8725	1919, Oct. 3	7.5	−25	5	2	1	
	C 130	Oct. 15	7.5	−13	4	2	2	
	C 137	Oct. 16	7.5	−12	6	3	3	
	C 1499	1921, Dec. 15	7.5	−16	12	5	5	
U Cassiop.	C 1449	1921, Nov. 12	8.2	− 4	8	4	4	
	C 1455	Nov. 13	8.2	− 3	9	4	3	
S Cassiop.	γ 10755	1922, Jan. 13	8.9	+22	7	2	.....	
T Camelop.	γ 11245	1922, Aug. 9	8.8	−36	5	2	.....	Obs. by Hoge
H.D. 35155	C 672	1920, Sept. 26	7.0	.....	.....	.....	.....	
R Gemin.	γ 8727	1919, Oct. 3	7.4	−27	8	3	0.5	
	C 127	Oct. 14	7.0	−16	10	5	2	
	C 133	Oct. 15	7.0	−15	12	6	2	
	C 144	Oct. 17	7.0	−13	12	6	2	
	C 681	1920, Sept. 28	7.2	−30	8	3	.....	
	C 821	Dec. 27	8.3	+60	10	4	0.5	
	C 1339	1921, Sept. 21	8.1	−50	4	0.5	.....	
	γ 10524	Oct. 10	7.2	−31	12	5	1	
	C 1402	Oct. 13	7.1	−28	.....	.....	.....	Red region
	C 1408	Oct. 14	7.1	−27	.....	.....	.....	Red region
	C 1456	Nov. 13	7.2	+ 3	15	7	2	
	C 1501	Dec. 15	7.3	+35	12	7	2	
	C 1502	Dec. 15	7.3	+35	.....	.....	.....	Green region
R Can. Min.	C 294	1920, Mar. 3	8.0	−10	15	5	0.5	
	C 299	Mar. 4	8.0	− 9	15	5	0.5	
	γ 9047	Mar. 7	8.0	− 6	8	3	.....	
	C 909	1921, Feb. 25	8.3	+20	15	7	2	
	γ 9990	Feb. 28	8.3	+23	.....	.....	.....	Red region
	C 1522	1922, Jan. 12	8.5	−33	6	3	.....	
T Gemin.	C 968	1921, Mar. 28	9.1	−23	2	0.5	0.5	
	C 1009	Apr. 29	8.6	+ 9	10	5	3	
H.D. 63733	C 1464	1921, Nov. 14	8.5	.....	.....	.....	.....	
V Cancri	C 194	1919, Nov. 9	8.0	+10	15	7	2	
	γ 8875	Nov. 13	8.0	+14	7	3	1	
	γ 8883	Nov. 14	8.0	+15	6	3	0.7	
	γ 10132	1921, Apr. 27	7.9	− 3	10	5	0.7	
	γ 10757	1922, Jan. 13	7.9	−12	9	5	0.7	
	C 1552	Feb. 11	7.8	+17	15	8	3	
S Urs. Maj.	C 369	1920, Apr. 8	8.5	−31	7	3	.....	
	C 373	Apr. 10	8.5	−29	11	5	.....	
	γ 9252	June 3	8.2	+25	6	3	.....	
	γ 9848	Dec. 29	8.4	+25	6	4	1	
	C 866	1921, Jan. 28	8.8	−52	4	2	.....	
	C 1553	1922, Feb. 11	8.5	−28	7	3	.....	
	C 1626	Mar. 19	7.6	+ 8	9	6	1	
H.D. 121447	C 1706	1922, May 16	8.1	.....	.....	.....	.....	
R Camelop.	γ 8266	1919, June 9	8.2	+ 7	4	2	.....	
	γ 8272	June 10	8.2	+ 8	2−	1	.....	
	γ 8345	July 8	8.9	+36	2	1	.....	
	γ 11077	1922, May 17	8.2	−18	4	2	0.4	
H.D. 172804	C 472	1920, June 6	9.1	.....	.....	.....	.....	Obs. by Sanford

TABLE II—*Continued*

Star	Plate	Date	Mag.	Phase	Intensities			Remarks
					Hβ	Hγ	Hδ	
				days				
T Sagittarii	C 1059	1921, June 20	8.5±	+39	7	3	1	
		1922, July 13	9.1	+65	6	2	.....	
R Cygni	C 965	1921, Mar. 27	7.3	+31	15	6	2	
	C 966	Mar. 27	7.3	+31	.....	.....	.....	Red region
	C 1007	Apr. 28	8.9	+63	15	5	2	
	C 1038	May 26	9.2	+91	15	5	2	
	γ 11079	1922, May 17	6.8	+ 4	10	4	1	
	γ 11117	June 14	8.0	+32	15	5	2	
	C 1784	July 11	8.9	+59	15	6	2	

spectrographs with 18-inch camera-lens foci, and are of the usual photographic region unless otherwise noted in the remarks. The γ plates were secured with the 60-inch reflector; the C plates with the 100-inch Hooker reflector. The column headed "Phase" gives the number of days before (−), or after (+), the nearest maximum. The intensities of the bright hydrogen lines are estimates of the strength of the lines on the negatives, no allowance having been made for instrumental or other observational effects. Hβ is usually overexposed and the estimates of this line are doubtless only rough approximations.

X ANDROMEDAE, H.D. 1167

Remark in *H.D.:*

On a photograph taken November 8, 1904, the spectrum was estimated Mb, having Hγ and Hδ bright, and equal in intensity.

The Mount Wilson plate taken on November 13, 1921, shows a spectrum similar to that of R Andromedae (C 137), but resembling still more closely R Geminorum (C 133) and S Ursae Majoris (C 373). The M bands are not recognizable and must be very weak, if present at all. Of the bright hydrogen lines, Hδ is not seen, Hγ is a fairly strong narrow line, and Hβ is overexposed. The intensities of Hβ and Hγ are perhaps as six or eight to one. This estimate is best made from the plate taken on the following night, which has much less exposure, and on which the strong bright Hβ line is the only outstanding feature; Hγ is clearly seen but is very weak.

It is evident that at the 1921 maximum the spectrum differed very greatly from that at the date of the Harvard observations which was less than a month after the 1904 maximum. Changes of this character are of the highest interest as bearing on the relationships between classes M and S. It is hoped to discuss them in a subsequent contribution, after more observational data have been secured.

R ANDROMEDAE, H.D. 1967

Remark in *H.D.:*

The spectrum is very peculiar, and does not appear to be of any division of Class M. In the continuous portion between $H\beta$ and $H\gamma$ it resembles the spectrum of π^1 Gruis, R.A. $22^h16^m.6$, Dec. $-46°27'$, and should probably be placed in some division of Class R. The continuous portion of the spectrum is very faint in the region having shorter wave-length than $H\gamma$. The hydrogen lines $H\beta$, $H\gamma$, $H\delta$, and $H\zeta$ are bright. $H\beta$ is about 0.3 as bright as $H\gamma$ and $H\delta$ which are nearly equal.

If we applied to S-type stars the criteria of absolute magnitude developed by Adams and Joy for K and M stars, R Andromedae would appear to be of high luminosity (perhaps comparable with K and M giants), since λ 4215 is strong and λ 4455 weak. The bright hydrogen lines are very conspicuous. The Harvard and the Mount Wilson observations agree in making bright $H\zeta$ stronger than $H\epsilon$. This circumstance is discussed on page 471. Several narrow maxima are present in the continuous spectrum, some of which, at least, are doubtless true emission lines. Among these are the enhanced iron lines $\lambda\lambda$ 4584, 4924, 5018. (See page 472.)

U CASSIOPEIAE, H.D. 4350

Remark in *H.D.:*

The spectrum resembles that of R Andromedae, H.D. 1967, and does not appear to be of Class M. $H\gamma$ and $H\delta$ are bright. On a photograph taken July 10, 1894, these two lines are of equal brightness, but on photographs of October 8, 1896, and December 1, 1905, $H\delta$ is from 3 to 6 times as bright as $H\gamma$.

The spectrum shows considerable difference from that of R Andromedae (C 137) and more nearly resembles that of R Geminorum (C 133). There is a definite indication of the titanium band at λ 4954, and possibly a trace of the one at λ 4761.

S CASSIOPEIAE, H.D. 7769

Remark in *H.D.:*

The spectrum is peculiar, and does not appear to be of Class M, but to resemble in characteristics that of R Andromedae, H.D. 1967. On several photographs examined the brightness of $H\beta$ varies from 2 to 10 times that of $H\gamma$.

On the Mount Wilson spectrogram, γ 10755, bright $H\beta$ is several times as strong as $H\gamma$. The continuous spectrum is very weak but traces of the principal S features are seen.

T CAMELOPARDALIS, H.D. 29147

Remark in *H.D.:*

The spectrum resembles Class R, in having the strong absorption band 4640–4750, and other dark bands between $H\beta$ and $H\gamma$. It is very faint in the region of shorter wave-length than $H\gamma$.

The only Mount Wilson plate is underexposed but several of the chief S features can be identified.

R ORIONIS, H.D. 31798

Remark in *H.D.:*

The line $H\beta$ is bright. The spectrum shows strong dark bands and may belong to Class R.

H.D. 35155

Remark in *H.D.:*

The spectrum resembles that of π^1 Gruis in the region from $H\beta$ to $H\gamma$.

Omitting the bright hydrogen lines, this spectrum is on the whole a very good match for that of R Geminorum (C 133), although a few of the less conspicuous absorption lines are weaker than in R Geminorum and there are slight differences in the characteristic S structure near λ 4650. The titanium bands are more prominent, that at λ 4954 being fairly strong, with indications of others.

R LYNCIS, H.D. 51610

Remark in *H.D.:*

The spectrum does not appear to be of Class M, but to resemble that of R Andromedae, H.D. 1967. The lines $H\beta$, $H\gamma$, and $H\delta$ are bright. On photographs taken November 27, 1905, and December 12, 1907, the line $H\gamma$ was the strongest bright line. $H\beta$ and $H\delta$ were respectively 0.4 and 0.1 as strong as $H\gamma$.

The Ann Arbor observations indicated that this spectrum is similar to that of R Cygni as observed by Wright. See *Publications of the Observatory, University of Michigan*, **2**, 52, 65, 1916.

R GEMINORUM, H.D. 53791

Remark in *H.D.*:

The spectrum is peculiar and resembles that of Class R rather than Class M. Two dark bands are present between $H\beta$ and $H\gamma$. On photographs taken October 25, 1906, and October 12, 1910, $H\gamma$ is bright. The spectrum is not seen in regions of shorter wave-length than 4300.

As compared with R Andromedae there is a general similarity but many differences are obvious. Bright $H\delta$ is much weaker relatively to $H\gamma$ than in R Andromedae. The continuous spectrum does not extend as far toward the violet. The absorption line λ 4554 is much stronger and broader. There are numerous differences in the relative intensities of lines and in the details of band structure. The spectrograms of the red region record $H\alpha$ as a bright line, and show the strong absorption band at about λ 6470, which was photographed by Wright in the spectrum of R Cygni (*Monthly Notices*, **72**, 548, 1912).

R CANIS MINORIS, H.D. 54300

Remark in *H.D.*:

The spectrum is peculiar and resembles that of R Andromedae, H.D. 1967. On photographs taken February 3, 1897, February 1, 1910, and January 24, 1911, the line $H\gamma$ is bright.

The spectrum differs somewhat from that of R Geminorum, and decidedly from that of R Andromedae. As compared with R Geminorum (C 133) there are some dissimilarities in the details of the band absorption near λ 4645, and a feature at λ 4736 is quite different. The plate of the red region is badly underexposed but shows the $H\alpha$ line to be bright.

H.D. 58881

Remark in *H.D.*:

The spectrum is not seen in the region of shorter wave-length than 4200. It resembles that of π^1 Gruis. Bright and dark bands are situated between $H\beta$ and $H\gamma$.

T GEMINORUM, H.D. 63334

Remark in *H.D.:*

On a photograph taken February 8, 1893, the spectrum is peculiar and probably resembles that of R Andromedae, H.D. 1967. It is very faint except in the region between $H\beta$ and $H\gamma$. The lines $H\gamma$ and $H\delta$ are bright.

The spectrum is much like that of R Geminorum, though not an exact duplicate.

H.D. 63733

Remark in *H.D.:*

In the region of 4700, the spectrum resembles that of π^1 Gruis, R.A. $22^h16^m.6$, Dec. $-46°27'$. It is very faint from $H\gamma$ to the end of shorter wave-length.

Except for the bright hydrogen lines, which are missing, this spectrum is a rather close match for that of R Geminorum (C 133).

V CANCRI, H.D. 70276

Remark in *H.D.:*

On a photograph taken March 30, 1892, the lines $H\beta$ and $H\gamma$ are nearly equally bright, and the spectrum is very faint from $H\gamma$ to the end of shorter wave-length. The spectrum may resemble that of R Andromedae, H.D. 1967.

This spectrum is much like that of R Geminorum, but the band absorption is slightly weaker.

S URSAE MAJORIS, H.D. 110813

Remark in *H.D.:*

On photographs taken February 22, 1906, and April 26, 1909, the spectrum is of a peculiar class, and resembles that of R Andromedae, H.D. 1967. The line $H\gamma$ is bright. A bright band is situated at about 4645.

This spectrum is much like that of R Geminorum. Certain changes dependent on the light phase have been observed and will be described in another contribution.

H.D. 121447

Remark in *H.D.:*

The spectrum is not seen in the region having shorter wave-length than 4200. It bears some resemblance to the spectrum of π^1 Gruis between $H\beta$ and $H\gamma$.

With the exception of the bright hydrogen lines, which are missing, this spectrum is a fair match for that of R Geminorum

(C 133), although there are some differences in the relative intensities of certain features. The band absorption is weaker.

R CAMELOPARDALIS, H.D. 127226

Remark in *H.D.:*

On a photograph taken May 20, 1890, the spectrum is peculiar with bright and dark bands between $H\beta$ and $H\gamma$, and resembles that of R Andromedae, H.D. 1967. The lines $H\beta$, $H\gamma$, and $H\delta$ are bright. The relative intensities are 2, 10, and 2.

All of the Mount Wilson plates are underexposed. The continuous spectrum is too faint to classify with certainty, but it is probably of class S.

RW LIBRAE, H.D. 136734

Remark in *H.D.:*

On a photograph taken June 3, 1907, the spectrum has the lines $H\beta$, $H\gamma$, and $H\delta$ bright. The intensities are 8, 10, and 3, respectively. The continuous spectrum is much stronger between $H\beta$ and $H\gamma$ than in other portions. It is not well defined but probably resembles that of R Andromedae, H.D. 1967.

H.D. 172804

Remark in *H.D.:*

The spectrum is faint. It may resemble that of the variable star 043065, T Camelopardalis, H.D. 29147.

With the exception of the bright hydrogen lines, which are missing, this spectrum closely resembles that of R Geminorum, but there is more contrast in certain features. The band absorption is probably stronger than in any other non-variable S-type star observed at Mount Wilson. Parts of the spectrum are much like that of π^1 Gruis, but a band-head near λ 4736 is lacking. The plate of H.D. 172804 was secured by Mr. Sanford in connection with his program for class R stars. I am indebted to him for permission to make use of it.

T SAGITTARII *191017*

Remark in *H.A.*, **56**, page 211:

The spectrum changes from Md0 to Md4, on 8 plates. $H\delta$ apparently changes from 30 to 70. On two plates the spectrum is peculiar, and on one plate it is Class Md peculiar.

On the Mount Wilson plates the continuous spectrum is too faint to classify with certainty but is probably of class S.

R CYGNI 193449

Remark in *H.A.*, **56**, p. 211:

The spectrum of this star is peculiar, and shows changes on the various photographs.

This spectrum resembles that of R Geminorum (C 133) more than that of R Andromedae (C 137), but has the band absorption somewhat stronger than in R Geminorum. Photographs of the spectrum of R Cygni were secured by Wright in December, 1911, and are described in the *Monthly Notices*, **72**, 548, 1912. The Mount Wilson plate of the red region is underexposed, but shows a strong bright $H\alpha$ line and the prominent band at about λ 6470 as observed by Wright. Visual observations by Espin, recorded in the *Monthly Notices*, **72**, 546, 1912, indicate that during the latter part of 1911 the spectrum varied as the brightness increased. The recent Mount Wilson photographs, including some not listed in Table II, also show very considerable changes in some features of the spectrum. They are briefly described in the *Publications of the Astronomical Society of the Pacific*, **33**, 206, 1921. A more complete discussion of the changes is reserved for another paper.

π^1 GRUIS

Remark in H.A., **56**, page 219:

The spectra of T Camelopardalis and π^1 Gruis resemble each other and are very peculiar. The absorption band between 4640 and 4750, is present, and almost all of the continuous spectrum between this band and $H\gamma$ is cut out by other strong absorption bands.

This star is too far south for the Mount Wilson telescopes, but slit spectrograms taken at the Lick Observatory station at Santiago, Chile, in July and September, 1915, have been made available to me through the kindness of Dr. Campbell and Dr. Moore. This spectrum is the only one thus far observed with slit spectrographs which resembles R Andromedae more than R Geminorum. Several bright lines or maxima are weaker in π^1 Gruis than in R Andromedae, but on the whole the similarity is very close, with, of course, the

exception of the bright hydrogen lines which are seen only in R Andromedae. The general similarity of the absorption-band spectra of these two stars was clearly brought out by Miss Cannon through a comparison of the Harvard objective-prism plates. See *Publications of the American Astronomical Society*, Twenty-third Meeting, Ann Arbor, 1919.

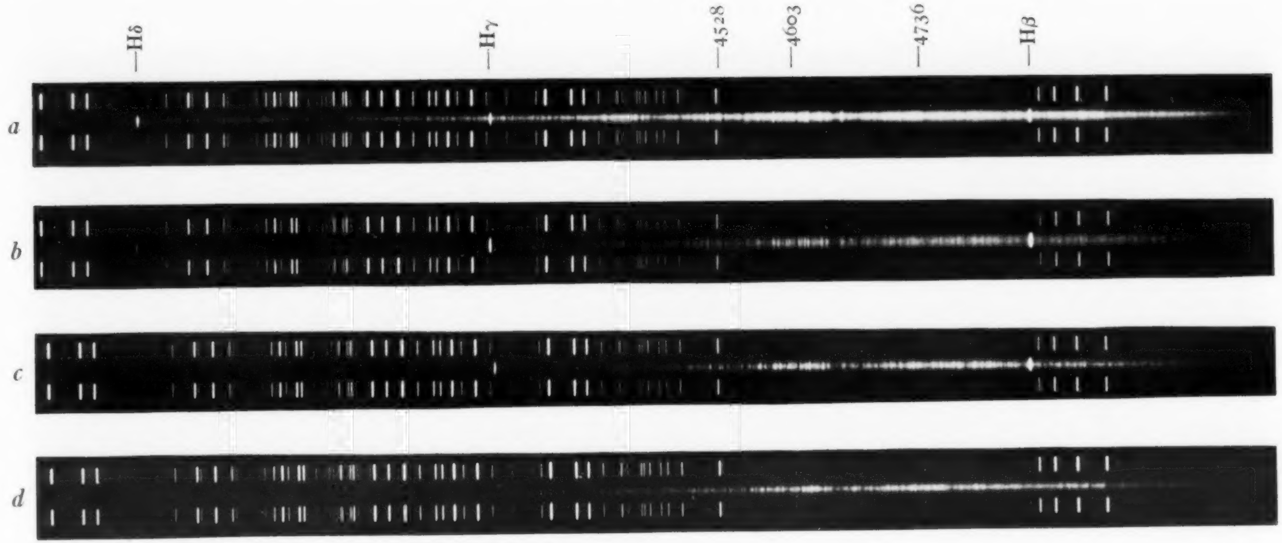
GENERAL DISCUSSION

The resemblance between classes R and S which appears from objective prism observations, is seen by means of slit spectra to be of the most general character only. The band in the blue is of decidedly shorter wave-length in class S, and there is throughout this region a marked lack of correspondence in the spectral details. On the other hand some of the characteristic S features are recognizable in the spectra of many giant stars of classes K and M, being more strongly developed in some stars than in others. From the comparisons which have thus far been made it appears that in general S stars are more closely related to giant Ma stars than to those of other types.

Perhaps the most characteristic feature of the S spectrum is a complicated structure in the region λ 4630 to λ 4660. This apparently consists of both absorption and emission lines and probably also contains one or more band-heads in absorption. A fairly definite idea of this portion of the spectrum may perhaps be gained by comparing the illustrations with the data in Table III. An absorption band in the red with its head near λ 6470 and extending toward longer wave-lengths is probably characteristic of class S, and may prove a useful criterion for classification.

The wave-lengths in Table III depend upon measurements of one-prism spectrograms of several variables of class Se. In the second column, *a* denotes absorption line, *e*, emission line or maximum, *bh*, band-head. The values given in the table were computed upon the assumption that the *e* features have the same velocity displacement as the bright hydrogen lines, and that they are systematically 0.2 A toward the violet from the absorption lines. Thus the emission lines and maxima were corrected for the apparent velocities obtained from the bright hydrogen lines, while

PLATE XIV



ONE-PRISM SPECTROGRAMS OF STARS OF CLASS S

(*a*) R Andromedae, October 16, 1919.
(*b*) R Geminorum, October 15, 1919.
(*c*) S Ursae Majoris, April 10, 1920.
(*d*) H. D. 35155, September 26, 1920.

TABLE III

CHIEF FEATURES IN CLASS S SPECTRA λ 4500–λ 4860

I.A.	Character	Intensity	Identification
4506.6	*a*	3	....
11.4	*e*	var.	....
12.8	*a*	2	Ti?
21.4	*e*	var.	....
23.1	*a*	2	Ba?
27.4	*a*	2−	Ti?
35.5	*a*	3	Ti+?
40.3	*a*	1−	....
54.0	*a*	5	Ba
62.9	*a*	1	....
68.9	*e*	....	....
71.3	*a*	2	Mg+?
80.6	*a*	1	Va
83.6	*e*	var.	Fe Enh.
86.2	*a*	2	V
88.3±	*e*	....	....
94.3	*a*	2	V
4606.8	*a*	3	Sr+?
08.7	*e*	....	....
13.1	*a*	1	....
14.9	*e*	....	....
16.1	*a*	2−	Cr?
18.2 }	*e*	....	....
19.2 }	*bh*	....	....
20.2	*a*	2	....
31.5	*e*	....	....
34.1	*a*	2	....
36.6 }	*e*	....	....
37.5 }	*bh*	....	....
44.3	note	....	....
52.4±	*e*	....	....
75.1	*a*	1	Ti?
88.0	*a*	2	....
4707.1	*a*	1	....
09.9	*a*	1	....
58.6	*a*	1	....
84.1	*a*	1	....
99.8	*a*	2	Ti?
4811.7	*a*	1−	....
15.6	*a*	1	....
23.6	*a*	1−	Mn?
28.0	*a*	2	....
32.0	*a*	2	....

NOTES TO TABLE III

4511.4 *e*. This has been observed as a bright line in the spectra of four Md variables.
4521.4 *e*. This has been observed as a bright line in the spectrum of one Md variable.
4644.3. Center of broad absorption space.
4652.4 *e*. Broad or double.
4828.0 *a*, 4832.0 *a*. These lines are very persistent although of moderate strength.
4448.2. A maximum, possibly a bright line, has been measured in this position on twelve plates.

an additional correction of −0.2 A was given to the absorption lines. The need of this correction was indicated by measurements of a few plates in the region λ 4100–λ 4500. The relative displacement of bright and dark lines is analogous to that well known in Md spectra. The proper amount of the correction for the S spectra is possibly somewhat greater than 0.2 A but the data are too meager to yield a precise value. In one N variable Moore[1] found a corresponding displacement of 0.2 A. In Md variables it appears to be a function of the light period, being 0.25 to 0.3 A for the average period of the Se stars.

Continuous spectrum.—These stars are red, the color-indices of the variables probably being as great as that of the Md variables, namely, 1.8 magnitudes. As compared with Ma stars, the continuous spectrum falls off toward the violet from about λ 4500, and with greater rapidity after λ 4430.

Absorption lines.—Table III shows the chief absorption lines in the region λ 4500–λ 4860. The strength of the barium line λ 4554 makes it an outstanding feature. This line and λ 4607 (Sr) are much stronger than in classes K and M. The strength of λ 4496, compared to the neighboring line, λ 4494, is also notable. Only two or three spectrograms are available for the study of the region from λ 4450 toward the violet. Judged from these, the absorption spectrum in this region is more like that of class M than at the longer wave-lengths. In several portions of the spectrum there is, in fact, a considerable degree of correspondence between absorption lines in classes S, M, and N, in spite of their notable differences.

Bright lines.—The wave-lengths marked *e* in Table III refer to rather broad[2] maxima which vary in prominence in different spectra (especially λλ 4511.4, 4521.4, and 4583.6). In some spectra they are scarcely distinguishable from portions of continuous spectrum lying between absorption lines or spaces; in other spectra they appear to be outstanding bright features, decidedly above the general intensity of the neighboring continuous spectrum. Thus, while the designation *e* in the table may be considered merely as

[1] *Lick Observatory Bulletin*, **10**, 160, 1921.

[2] Exception should be made for λ 4511.4 and λ 4521.4, as these wave-lengths correspond to fairly narrow lines, the former appearing nearly monochromatic.

describing the more conspicuous maxima, it is believed by the writer that they probably correspond to actual emission. This is borne out for λ 4583.6, which is identified with the enhanced iron line λ 4583.8, by the presence of similar lines at $\lambda\lambda$ 4924, 5018, corresponding to other well-known enhanced iron lines.

As in class M[1], stars showing bright hydrogen lines are long period variables. The bright hydrogen lines have quite different properties, however, in the two classes. In class S the less refrangible lines are stronger, but they rapidly decrease in intensity in passing toward the ultra-violet.[2] The relative intensities of $H\beta$, $H\gamma$, and $H\delta$ are shown in Table II.[3] The lines to the violet of $H\delta$ are usually too weak to be observed, even when $H\beta$ is strongly overexposed. In fact, $H\epsilon$ has never been seen on the Mount Wilson plates of S spectra. In one or two stars, however, the bright $H\zeta$ line has been recorded here and also at Harvard. Thus, in these cases at least, $H\zeta$ is stronger than $H\epsilon$. As is well known, this is also true in Md stars. It is particularly interesting to find it so in S spectra, where the Balmer series has widely different characteristics, as this circumstance lends support to the not wholly acceptable conclusion that $H\epsilon$ is actually emitted, but subsequently weakened through the absorption of overlying calcium vapor. $H\alpha$ is bright on the few plates of the red region thus far secured.

[1] Certain M stars recently found to have bright lines either do not vary in magnitude, or vary irregularly through a small range. Some of these stars are dwarfs (Adams and Joy, *Publications of the Astronomical Society of the Pacific*, **32**, 158, 1920, and **34**, 174, 1922), and others (*ibid.*, **34**, 175, 1922), while evidently giants, have spectra differing markedly from that of *o* Ceti and other well-known Md variables. Considering only stars whose underlying spectrum is similar to that of the typical M giants, we find what seems to be a perfect correspondence between variability and the presence of bright hydrogen lines.

[2] This is also true of the hydrogen lines in the peculiar M stars referred to in the preceding footnote, but, as these lines are reversed or somewhat widened, their appearance is different from the narrow hydrogen lines in S stars. See *Publications of the Astronomical Society of the Pacific*, **34**, 175, 1922. As observed with one-prism dispersion, the $H\gamma$ and $H\delta$ lines of S spectra appear monochromatic. On most of my negatives $H\beta$ is too strong to judge of its real character. It may be slightly widened.

[3] Comparable estimates of the intensities of the bright hydrogen lines in Md spectra may be found in Table III, *Mt. Wilson Contr.*, No. 200; *Astrophysical Journal*, **53**, 191, 1921.

In comparing the relative intensities of the bright hydrogen lines as given by remarks in the Harvard publications with those of the Mount Wilson plates as shown in Table II, there seem to be many striking differences, but allowance must be made for the fact that in many cases the Harvard observations were made with emulsions much less sensitive to the $H\beta$ region (relatively to $H\gamma$ and $H\delta$) than those now in use. I do not know whether or not a difference in focus between these lines on the Harvard plates could have effected the apparent relative intensities.

An unexpected feature is the presence of the enhanced lines of iron, $\lambda\lambda$ 4584, 4924, 5018. We have been accustomed to think of enhanced lines as being prominent in classes A to G, especially in the intrinsically brighter stars, and becoming inconspicuous in the later (redder) spectral classes. The presence of the above-mentioned lines in these red S variables, as well as the observations by Adams, Joy, and Humason of enhanced lines in certain peculiar M stars,[1] represents, in our present limited knowledge of stellar conditions, an anomalous circumstance which invites investigation.

The mean period of sixteen class S variables is 364 days. This is about the same as that of the Md stars having the more advanced spectra. The data for the Md stars in the following small table are taken from *Harvard Annals*, **76**, Table IX.

Class	Number	Period
Md 3, 4, 5	90	260
Md 6, 7	70	302
Md 8, 9, 10	67	355
Se	16	364

The recognition of the S stars as a distinct group may remove part of the difficulty in studying the relationship between spectral type and period among the long period variables.

The evolutionary relationship of class S stars to those of other classes, as well as the dependence of spectral modifications upon absolute luminosity, present highly interesting problems for future investigation.

[1] *Publications of the Astronomical Society of the Pacific*, **34**, 175, 1922.

It seems probable that the S stars form a third division of the giant sequence, and that they are related more closely to M than to N stars. Whether or not stars exist having spectra intermediate between Mb, Mc, and the advanced S type remains to be determined, but it is suspected that it may be possible at least to fill the gap between Ma and the early S spectra. A tentative notion of the spectral relationships among red stars is represented by the diagram in Figure 1.

Many questions are suggested which cannot profitably be discussed here. One noteworthy circumstance, however, may be pointed out, namely, that along each of the three branches, M, N, and S, there is an increasing tendency toward variability in light,

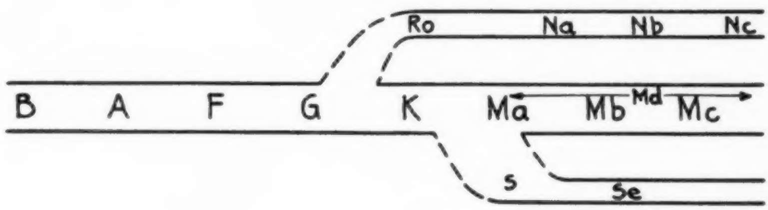


FIG. 1.—Hypothetical relationships of red stars

and toward the presence of bright lines in the spectra. Certain facts[1] might be interpreted as indicating that the general cause of the light variations is essentially the same in both Md and Se stars, with the possibility that the differences in spectrum are due either to differences in conditions which are not important in controlling the amount of light radiated, or to differences in chemical composition. Dr. Shane, who made a special study of N-type variables,[2] is of the opinion that the main causes underlying the light variations are probably common to both Md and N stars. It is not easy to say, however, just what generalizations may properly be drawn from the analogies between the M, N, and S branches. The effective temperatures are probably not the same,

[1] Observations of R Cygni bearing on this matter are briefly discussed in the *Publications of the Astronomical Society of the Pacific*, **33**, 206, 1921.

[2] *Lick Observatory Bulletin*, **10**, 79, 1920.

as several lines of evidence, especially the measurements of heat indices[1] by Nicholson and Pettit, indicate that the effective radiating temperature of the variables similar to *o* Ceti is decidedly lower than that of other red stars.

It is hoped to treat the radial velocities of Se stars, the red portions of the S spectrum, the variable spectra of certain S stars, and the general relationship between classes M and S in future contributions.

MOUNT WILSON OBSERVATORY
August 17, 1922

[1] *Publications of the Astronomical Society of the Pacific*, **34**, 181, 1922.

THE BEHAVIOR OF SPECTRAL LINES AT THE POSITIVE POLE OF THE METALLIC ARC[1]

By PAUL W. MERRILL

ABSTRACT

Classification of arc lines by relative strengthening at the positive pole.—In the Pfund type of arc there is a small well-defined region of luminous vapor just above the point where the core of the arc enters the molten bead which forms the positive electrode, and at the boundary of this region many spectral lines change suddenly in intensity to a greater or less degree. By keeping the core of the arc steady and focusing it on the slit of a 30-foot Littrow spectrograph, plates were obtained from which it was possible to group about 500 *iron lines* from λ 3849 to λ 5763 into six classes: change imperceptible (0), slight (1), small (2), sufficient to double the intensity (3), great (4), or very great (5). The *relationship to the furnace classification* of King was studied for each group. In general the strengthening at the pole is greater for the high-temperature lines, the average class for King's classes I, II, and III being, respectively, 0.2, 1.0, and 1.9. But the average for classes IV and V is only 1.8, and for the diffuse lines of these classes, only 1.3. Lines of each temperature class are also found in each of the positive-pole classes 0, 1, and 2; hence the correlation is only fair, although probably as close as can be expected with such different modes of excitation. The strengthening of the enhanced lines is very marked (average class being 4), so that positive pole observations may be useful in distinguishing these lines. The classifications of 180 *cobalt lines* and of some *nickel lines* show the same general relationships as were found for the iron lines.

An interesting recent development in spectroscopy has been the increasing importance attached to the classification of spectral lines according to their behavior under various conditions of excitation. This has been due in part to the desire to provide an improved basis for the interpretation of spectroscopic observations of various light sources, particularly of celestial objects, and in part to the wide discussion of the quantum and ionization theories of the emission of spectral lines.

It has long been recognized that the relative intensities of lines in light from different portions of the electric arc afford a means by which the lines may be grouped. Most of the spectroscopic investigations of the structure of the arc have been concerned with the carbon arc, but a number have dealt with arcs between metallic electrodes. Among the phenomena observed in metallic arcs, one of the most interesting and important is, that near the poles, especially the positive pole, lines appear which are absent or very weak in the

[1] *Contributions from the Mount Wilson Observatory*, No. 253.

center of the arc or in integrated arc light, but which are characteristic of the disruptive spark. Photographs made by the writer in 1919 exhibited this property of enhanced lines very clearly, and also showed systematic effects for other types of lines. The general results of a study of these photographs[1] are described in the present paper.

Iron was the element most extensively observed in this investigation. The arc used was of the Pfund type, the upper, negative electrode consisting of an iron rod, the lower, positive electrode of a drop of molten iron oxide. A copper rod hollowed out at the upper end served as a holder for the iron oxide and made possible an arc with a spheroid of oxide slightly smaller than usual. With currents of about two amperes the arc burned steadily with a small, sharply-defined, circular, bright area on the molten surface, from which the core of the arc proceeded. It proved important for the present purpose to have this positive "bead" localized and steady, for the vapors immediately surrounding it were the source of the special phenomena described in the following paragraphs. The general appearance of this portion of the arc is well shown in the illustrations accompanying papers on the iron arc by St. John and Babcock.[2]

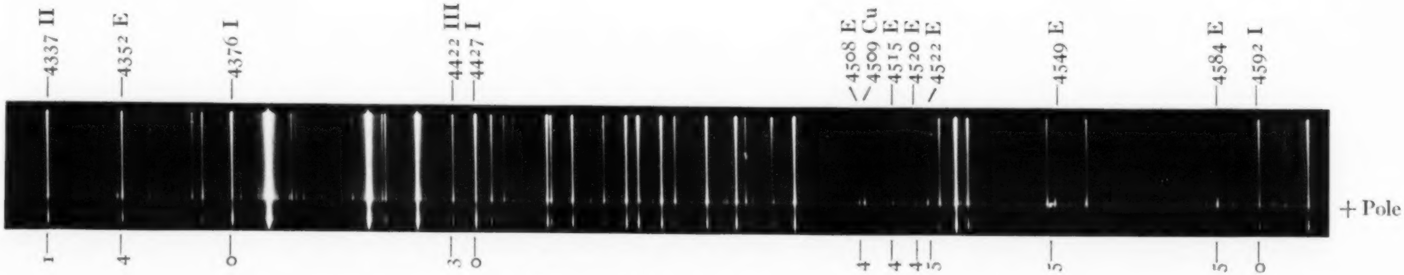
An image of the arc was projected in such a way that the line joining the electrodes coincided with the slit of the 30-foot Littrow spectrograph. In the photographs thus obtained each spectral line represents the emission of that radiation along the axis of the arc. The actual arc length was about 5 mm, the magnification of the image three times. Currents ranging from 1.5 to 3.2 amperes at 250 volts were employed.

The particular subject dealt with in the present paper is the increase in the intensity of spectral lines at the bright end of the arc core just off the positive pole. The effects at the positive pole are more sharply localized than those at the negative pole. Most lines show a continuous increase in intensity on passing from the

[1] Reported upon at the Pasadena Meeting of the Pacific Coast Section of the American Physical Society, June, 1919; *Physical Review*, **14**, 271, 1919.

[2] *Mt. Wilson Contr.*, Nos. 106, 137; *Astrophysical Journal*, **42**, 231, 1915; **46**, 138, 1917.

PLATE XV



Spectrum of the Iron Arc

Showing behavior of the lines at the positive pole. The figures above the spectrum indicate the wave-length and temperature class; those below the degree of strengthening at the positive pole. The extensions of the lines below the narrow strip of continuous spectrum marking the positive pole are due largely to reflection of the main portion of the arc from the molten spheroid of oxide forming the positive terminal.

center to the negative pole, but on approaching the positive pole, there is, for an affected line, a sudden increase in intensity at a point corresponding to the boundary of a small bright volume contiguous to the pole.

Nearly 550 iron lines between wave-lengths 3849 A and 5763 A were classified according to the increase of intensity at the positive pole on the following basis:

- 0 no increase
- 1 slightest observable increase
- 2 small increase
- 3 considerable increase (intensity about doubled)
- 4 great increase
- 5 very great increase

The relationship of this classification to other classifications of the same lines is exhibited in Table I, which gives for each class the number of lines of each degree of pole strengthening. Thus

TABLE I

POSITIVE POLE STRENGTHENING OF IRON LINES

Class	0	1	2	3	4	5	Total	Average
I	51	10	2	......	......	......	63	0.2
II	18	15	10	3	......	......	46	1.0
III	3	24	50	14	......	......	91	1.9
IV	4	29	65	21	3	......	122	2.0
IV*n*	3	16	15	......	......	......	34	1.4
V	......	15	24	2	......	......	41	1.8
V*n*	1	19	6	......	......	......	26	1.2
e	......	1	4	8	1	......	14	2.6
E	......	....	1	8	12	7	28	3.9

the numbers in the first row indicate that out of sixty-three class I lines fifty-one were not strengthened, ten were very slightly strengthened, and two showed a somewhat greater increase at the positive pole. The Roman numerals I–V refer to the temperature classes determined by King[1] from data obtained by use of the electric furnace; *E* indicates enhanced lines, and *e*, lines with a negative pole effect.[2] In compiling the numbers for this table it was noticed

[1] *Mt. Wilson Contr.*, Nos. 66, 247; *Astrophysical Journal*, **37**, 239, 1913; **56**, 318 1922.

[2] St. John and Babcock, *Mt. Wilson Contr.*, No. 202; *Astrophysical Journal*, **53**, 260, 1921.

that many of the lines which fell in columns 0–2 of rows IV and V were broad or nebulous. Accordingly all lines of these classes that are so marked, either by King[1] or Burns,[2] were separated and placed in rows IVn and Vn.

The chief facts brought out by this study may be seen by inspection of Table I. To facilitate comparison of the various types of lines, the average value of the pole strengthening for each has been placed in the last column. The progression in these average values is shown by the following grouping:

Class	I	II	IVn, Vn	III, IV, V	e	E
Number	63	46	60	254	14	28
Average strengthening	0.2	1.0	1.3	1.9	2.6	3.9

The e lines in the green, λ 5364–λ 5598, are of special interest. They are in furnace class V and are greatly strengthened at the positive pole. In confirmation of the observations of St. John and Babcock it was noted that at the positive pole these lines are distended toward shorter wave-lengths. The d lines in the regions λ 5232–λ 5624 are in class IV and in group 2 as regards pole strengthening.

The enhanced lines, designated by E, are from Lockyer's table, with a few additions from other sources. The degree of strengthening at the positive pole seems to be approximately proportional to the amount of enhancement in passing from arc to spark, but the data at hand do not allow any precise determination of this relationship. They suggest that there is a sequence of types of lines from I at one end to enhanced lines at the other; that classes I, II, and III follow in regular order, but that classes IV and V contain two types of lines, one type continuing the sequence I–III and connecting it with the enhanced lines, the other type similar to the nebulous lines in regard to furnace behavior and pole strengthening but not separated from the others by the present treatment of the data. If these remaining lines, which really belong to the subgroup represented by "n," were removed, it is possible that the pole values for the remainder of the group would rise, and perhaps take

[1] *Loc. cit.* [2] *Lick Observatory Bulletin*, **8**, 27, 1913.

an appropriate position between classes III and *E*. To make a complete separation of the classes IV and V into two subgroups would be rather difficult, because the characteristics of the two apparently overlap. That is to say, while subdivisions are indicated with definite differences between them, lines intermediate in their characteristics may not be lacking. It would appear rather

TABLE II

POLE STRENGTHENING OF INDIVIDUAL IRON LINES

λ Rowland	Furnace	Positive Pole	λ Rowland	Furnace	Positive Pole
3950.102	III	1	4260.640	III	2
3951.311	IV	2	4308.081	II	0
3966.212	III	2−	4291.630	IA	0
3966.778*n*	IV	2	4294.301	II	1
4100.901	IIA	0	4415.293	II	0+
4107.649	III	3−	4427.482	I	0
4177.698	IIA	0	4459.301	III	3
4181.919	III	3	4461.818	I	0
4199.267	III	2	4489.911	IA	0
4202.198	I	0	4494.738	III	1
4206.862	IA	0	4528.798	II	2
4210.494	III	2	4531.327	II	0
4216.351	I	0	4919.174	III	2
4219.516	IV	4	4939.868	I	0
4232.887	IA	0	5050.008	III	2
4233.772	III	1+	5051.825	I	0
4250.287	III	1	5166.454	IA	0
4250.945	II	0+	5171.778	II	1
4271.325	III	1+	5341.213	II	1
4271.934	II	1	5371.734	I	0

arbitrary to attempt a division on the basis of the pole data alone. Probably the division based on the sharpness of the lines could be carried farther. The results would be of value when correlated with pole data. The *e* lines are peculiar in that, although lacking in sharpness, they have high pole strengthening.

It may be of interest to cite several examples of individual lines showing different behavior at the positive pole. For this purpose I

have chosen pairs of lines in King's list of "Neighboring Lines Which Are Differently Affected in the Furnace."[1] This will be sufficient to exhibit typical behavior of the various classes, and also some of the common variations. Comparison with Table I will show which lines represent average values and which deviations. λ 4219.516 is included by Gale and Adams in the list of enhanced lines.

Cobalt.—Estimates of the strengthening of cobalt lines at the positive pole of the arc were made in the same manner as for iron. For the summary in Table III 180 lines between the wave-lengths 3845 and 4629 A were used.

The bright volume of vapor immediately above the spot on the positive pole in the cobalt arc is not so sharply localized as with iron, and more lines show a gradual strengthening toward the positive pole, so that the appearance of the photographs is slightly different and in some cases the estimates less definite than for iron. The main features of the phenomena are the same, however, and the mean values of pole strengthening for the different furnace classes are nearly the same as for iron.

TABLE III

POSITIVE POLE STRENGTHENING OF COBALT LINES

Class	0	1	2	3	4	Total	Average
I	25	12	2	....	....	39	0.4
II	2	15	8	4	....	29	1.5
III	....	11	45	9	....	65	2.0
IV	....	2	9	2	....	13	2.0
V	1	5	7	1	....	14	1.6
E	2	1	9	5	3	20	2.3

The enhanced lines in the last row are all of Lockyer's list which have been observed. Some of these lines which give small pole strengthening do not seem to be characteristic enhanced lines. If this row were restricted to well-marked cases of enhancement, a value of 3 or 4 would be found instead of 2.3.

Not many lines in the cobalt spectrum have been noted as hazy. Four class V lines of this character give a mean value of pole strengthening less than that of the whole class. Thus the meager data available agree with iron in this particular also.

[1] *Mt. Wilson Contr.*, No. 66; *Astrophysical Journal*, **37**, 239, 1913.

A smaller amount of data for *nickel* indicates that the same general relationships exist for this element also.

The fact that grouping the spectral lines of three elements according to their behavior at the positive pole of the arc gives a classification which runs parallel to the temperature classification is of interest in showing that the inherent differences of the various classes of lines are such as to allow them to be distinguished in sources other than those upon which the classification was based. The chief factor which differentiates the various classes of lines is, presumably, the degree of excitation required for their production. To what extent the physical conditions at the center and at the positive pole of the arc are identical with those in the furnace at low and at high temperatures is difficult to judge, but we may at least say that the relative effects upon the atom, as regards radiation, are comparable.

That the unsharp lines are subject to a smaller increase of intensity at the positive pole than are sharp lines of corresponding temperature classes is an outstanding fact for which a certain explanation is not now available. It may be that the unsharp lines represent vibrations lacking in stability, being easily perturbed, as shown by the lack of homogeneity, and that under the more intense conditions at the pole these vibrations can be executed by relatively few electrons.

The ease and definiteness with which enhanced lines can be distinguished at the positive pole suggest that arc observations should be given consideration in the formation of lists of enhanced lines. A particular advantage is that the criterion is not the strength at the pole, but the increase in intensity (amounting practically to a discontinuity) at a definite point in passing from the center to the positive pole. This can be shown by a single photograph, and the selection of enhanced lines should be less arbitrary than when based upon a comparison of the relative strength of lines in separate arc and spark exposures.

The type of observations described in the present paper is possible only on a steady arc, such as the Pfund form of iron arc. Several of the metals can be made to burn in this fashion, and perhaps

methods can be developed by which the arcs of numerous elements can be sufficiently controlled to permit the gathering of similar data. With proper precautions, valuable results can probably be obtained for certain elements by using iron electrodes containing small amounts of the elements in question.

At present, positive pole phenomena would seem to be most useful in their bearing on problems connected with the classification of spectral lines, but they may also prove valuable in studying the temperature and electrical conditions at different points in the arc.

MOUNT WILSON OBSERVATORY
August 1922

GEOMETRICAL PROOF FOR THE WADSWORTH CONSTANT DEVIATION SYSTEM[1]

By R. C. GIBBS and J. R. COLLINS

ABSTRACT

Axis of rotation for Wadsworth constant deviation system.—In order that there may be no lateral shift of the emergent ray, the Wadsworth constant deviation system should be rotated about the intersection of the plane of the mirror and the plane bisecting the angle of the prism, as an axis. *A geometrical proof of the location of the axis* is presented which is much shorter than the analytical one given by Wadsworth.

In the Wadsworth constant deviation system, a triangular prism and mirror are mounted on the same rotating table. If a parallel beam of white light is incident upon the prism so that some color passes through the prism at minimum deviation and is reflected by the mirror, then, as the prism and mirror are rotated together about *any* axis that is perpendicular to the principal plane of the prism and to the path of the incident beam, the path of the emergent beam for whatever color is passing through at minimum deviation remains fixed in direction.

Let OP represent the direction of the incident beam, PQ that of the refracted beam, and QD that of the reflected or emergent beam. By an analytical method, Wadsworth[2] determined the particular axis about which the prism and mirror may be rotated without producing a sidewise displacement of the emergent beam. Using the direction of the incident beam as one of his co-ordinate axes he assumed an axis of rotation as at R and determined in turn the equation of SQ, a line lying in the plane of the mirror, that of PQ, and that of QD. This enabled him to find the co-ordinates of D and to write an equation for the perpendicular distance, p, from the origin O to QD as follows:

$$p=2a\cos^2\beta/2+2b\sin\theta/2+2d\cos(\theta+\Phi),$$

[1] The proof here given in full was presented at the Toronto meeting of the American Physical Society in December 1921, and a brief abstract of it appears in the *Physical Review* (2), **19**, 4, 381.

[2] *Astronomy and Astrophysics*, **13**, 835, 1894.

where a is the perpendicular distance from R to the incident beam, b that from R to PS which is the bisector of the prism angle, and d that from R to the plane of the mirror.

It is readily seen that in order for p to remain constant b and d must both be equal to zero. Therefore, if the emergent beam is to suffer no shift, the system must be rotated about S, the intersection of the bisector of the angle of the prism and the plane of the mirror.

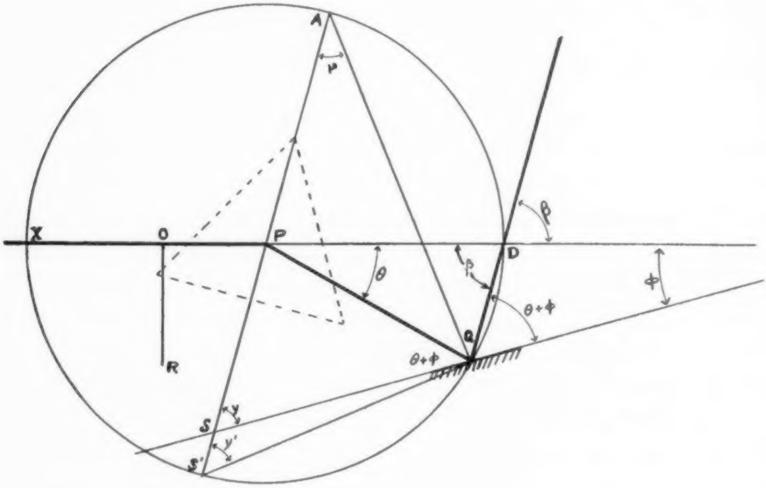


FIG. 1

Although the essential steps in this method are simple and straightforward, yet many of the transformations are necessarily long. By following a method similar to that used by E. Bloch[1] in finding the corresponding axis of rotation for a constant deviation prism of the Pellin-Broca type, the location of this axis may be determined by means of a short geometrical proof.

Describe a circle through D and Q having its center on PS, cutting PS at A and S' and the incident ray at X.

$$\text{Arc } XS' = \text{arc } S'Q.$$

[1] *Journal de Physique*, 7, 145, 1917.

Therefore $\beta/2$ is measured by $S'Q/2$ and $\mu=\beta/2$.

$$\gamma'=90-\beta/2$$

$$\gamma=180-\left(\frac{180-\theta}{2}+\theta+\Phi\right)=90-\left(\frac{\theta}{2}+\Phi\right).$$

$$\beta=\theta+\Phi+\Phi=\theta+2\Phi$$

Therefore

$$\gamma=90-\beta/2=\gamma'$$

and S' coincides with S. S therefore lies on the bisector of β for all positions of the rotating system.

Since β is constant and since one of its sides is the fixed incident ray, the bisector of β remains fixed in direction.

If then the system is rotated about an axis through S, the bisector of β will not shift and consequently the emergent ray will not shift.

There is no other axis of rotation for which this condition is satisfied, for, if the system is rotated about any other axis than the one through S, S will shift. Hence the bisector of β would shift and therefore the emergent ray would shift. By imposing other conditions for drawing the circle, it is possible to reach the same conclusion by other slightly different though similar procedures.

Cornell University
December 1921

REVIEWS

The Telescope. By LOUIS BELL. New York: McGraw-Hill Book Co., 1922. 8vo, pp. ix+287, figs. 190, plate 1. $3.00 (English price, 15*s*.) net, postpaid.

There are several small handbooks on the telescope, but the writer knows of nothing in English which brings together in compact form so much of information, suggestion, and helpful advice for the users of telescopes as are to be found in Dr. Bell's book. The author is well qualified for the preparation of such a book, being widely known for his studies of various optical as well as electrical problems. His purpose, as stated in the Preface, is "to bring the facts regarding the astronomer's chief instrument of research somewhere within grasp and up to the present time." Some of the material brought together here would not be readily accessible to the ordinary reader, and the references given at the close of a few of the chapters will be of value to those who wish to make a further study of the subjects treated. Similar lists of references might well have been added to other chapters.

The frontispiece is a fine engraving of Galileo's telescopes. The first two chapters trace the evolution of both refracting and reflecting telescopes, with a discussion as to their relative merits. The author dismisses with little comment the claim recently made that Bacon used telescope apparatus, although he admits that the great philosopher had clear ideas of refraction and that suggestions in his manuscript may have led to the making of spectacles, which were common long before the combination of lenses necessary for the refracting telescope was discovered. The processes of making, testing, and working optical glass are next described, the object being to give to the reader "a better understanding of the difference between the optical glass industry and the fabrication of commercial glass, and . . . . a fuller realization of how fine a work of art is a finished objective or mirror as compared with the crude efforts of the early makers or the hasty bungling of too many of their successors."

The fourth chapter deals with the properties of objectives and mirrors. The problems of spherical and chromatic aberration are considered, and the conditions that must be fulfilled if both are eliminated. A glance at the diagram of the spectrum (fig. 60), illustrating the irra-

tionality of dispersion, should make clear to any reader what is accomplished by combining lenses of crown and flint glass.

The consideration of various kinds of eye-pieces, with the advantages and disadvantages of each, is postponed until after a discussion of telescope mountings. Several varieties of mountings are described, from the simplest altazimuth for a small portable to the adaptation of the "English equatorial" mount used for the Hooker 100-inch telescope on Mount Wilson. Accounts are also given of special forms of mounting so designed that the observer may be protected, as the Gerrish polar telescope at Harvard, the Porter polar reflector, and the Hartness turret telescope. The amateur who has mechanical ability and wishes to make a mounting for his own instrument will find many suggestions here.

A few pages are devoted to hand telescopes and binoculars, special attention being given to the use of Porro prisms for field glasses and binocular telescopes.

In a chapter on accessories to the telescope we find described diagonal eye-pieces for observation of sun and stars, different kinds of micrometers and photometers. The necessity of a good clock drive whenever micrometer measures are to be made or photographs taken is emphasized and several forms are explained, the Gerrish electric control used successfully at the various Harvard stations being specially noted.

The last pages of the book are filled with practical suggestions. Excellent advice is given to the prospective purchaser of a telescope, and explicit directions for the testing of a small lens or mirror. The description of the Hartmann and Foucault tests will give the amateur an insight into methods used for larger instruments. The observer whose object glass needs a thorough cleaning will find here such detailed directions that he need not fear to remove his lenses from their cell, while methods of re-silvering mirrors are given for the owner of a reflector. The directions for adjusting the polar axis of a small equatorial could be followed easily, and the diagram (fig. 170) showing the position of the pole throughout the twentieth century with reference to Polaris and the two stars BD 88° 112 and BD 89° 13 is especially good. Various devices for protecting the telescope are described, from the simplest possible removable covering up to the revolving dome usually found in a fixed observatory.

The suggestions in the Appendix as to what may be done by a faithful observer with a very modest telescopic equipment are for the most part good, but surely the author does not mean to imply that the unknown rotation period of the planet Neptune might be found in this way!

The volume is a neat and attractive one, the type is good, the illustrations numerous and excellent. The author's statements are usually clear, but occasional obscure sentences occur and a number of slips and typographical errors mar the book. For instance, "A *seventh* century astronomer and his telescope" (see p. 15) would be a decided anachronism, and on page 204 the figure labelled "Ordinary striae" has evidently been interchanged with the one illustrating "A bad case of striae." Such errors will undoubtedly be corrected when a new edition is published.

A. S. Y.

Geschichte und Literatur der veränderlichen Sterne. By G. Müller and E. Hartwig. Leipzig: Poeschel & Trepte. Band II, 1920, 4to, pp. 468; Band III, 1922, 4to, pp. 137.

The first volume of this important publication of the Astronomische Gesellschaft appeared in 1918, and was reviewed at length in this *Journal*, **49**, 286, 1919. That review applies equally to the second volume, which includes the stars from right ascension 15^h to 23^h, thus completing the first part of the work. There are two appendices: the first, written by E. Zinner and G. Van Biesbroeck, giving the history and literature of thirty-two Novae which appeared between the years 1572 and 1912; and the second, written by C. Hoffmeister, dealing with Variables in Clusters.

The third volume contains the Catalogue, which arranges in form for ready reference the essential items from the earlier volumes. The data for each star occupy twenty-two columns of the double quarto page, giving the place for 1900 and the epoch of the chart, precession, color and spectrum, magnitude at maximum and minimum, calendar and Julian date of maximum epoch, mean period with interval from minimum to maximum, class of variation, volume in Hagen's *Atlas*, discovery name and date, and remarks. An appendix gives corresponding data for 320 variables and fifteen novae confirmed between 1915 and 1920. There is also an index of variables arranged by constellations, a table of light-equation for 276 variables of short period, a table of Julian days from 1600 to 2000, and a table for conversion of hours and minutes to decimals of a day.

This work is indispensable for astronomical libraries and observers of variable stars.

The price to American buyers is \$4.50 each for Volumes I and II; \$3.60, for Volume III. Members of the Gesellschaft are given a discount of one-third from these prices.

J. A. Parkhurst

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